

Impact of mean user speed on blocking and cuts of streaming traffic in cellular networks

Bartłomiej Błaszczyszyn¹ and Mohamed Karray²

Abstract—This paper deals with the performance evaluation of wireless cellular networks serving streaming traffic. In contrast to data traffic, streaming users require predefined transmission rates, which can be maintained at the price of blocking of some arrivals or cutting some existing connections when a network congestion occurs. The fractions of blocked and cut transmissions in the long run of the system, called respectively, blocking and cut probabilities, are the main performance metrics of such networks. In this paper we evaluate the impact of the intra- and extra-cell user mobility on these metrics. Specifically, we assume a spatio-temporal Poisson arrival process of streaming calls, independent Markovian mobility and exponential duration of each call. The dynamics of this free (offered) process is modified each time a congestion-generating transition occurs. We consider two possible modifications, which lead to two different loss models. We study both of them, and in particular, we propose some explicit approximations for the blocking and cut probabilities, which take into account the mean user mobility speed. We illustrate our approach studying UMTS release 99 building upon the scalable admission control schemes developed in [1], [2], which allow for an exact representation of the geometry of interference in the network.

I. INTRODUCTION

Cellular networks provide streaming services (voice calls, video streaming, etc.), which require predefined transmission rates, and carry elastic (data) traffic, which accepts fluctuations of the rates. In this paper we focus on streaming that represents an important fraction of the traffic in real life networks. The fixed rates of streaming can be maintained in a shared medium at the price of blocking of some arrivals or cutting some existing connections when a network congestion occurs. Obviously, the fractions of blocked and cut connections, which are important performance metrics for such systems, should be kept small, typically $\ll 10\%$. Analytical study of these metrics can help to satisfy this objective when planning and/or dimensioning of the network. The general objective of this paper is to *propose a simple, yet adequate method for the evaluation of blocking and cut probabilities in cellular networks with streaming mobile traffic*. In particular we want to capture the impact of the user mobility speed on these performance characteristics.

In order to identify congestion epochs, which may lead to blocking or cuts of some users in wireless communication networks, one has to take into account the geographical locations of users and the configuration of powers allocated to them. These powers create the interference at the receivers and thus determine the feasible transmission rates. Assuming geographical user mobility essentially complicates this task.

In particular, a call (voice call, video streaming, etc.) can be rejected (blocked) when it is arriving to the network but also prematurely terminated (cut) when the user moves. This latter can happen not only when the mobile changes its cell (*handoff*) but in principle also even if it only moves away from the serving base station while staying in the same cell. One may reasonably expect that longer distances traversed by mobiles during the calls (which is obviously related to the user speed) will cause more call cuts. On the other hand, cutting calls in progress reduces the global charge of the network and presumably increases the chances of new arrivals to be accepted. In this paper we verify this reasoning and try to *explicitly quantify the dependence of blocking and cut probabilities on the mean user mobility speed*.

We already said that in wireless networks it is the geographic location of mobile users that determines the joint feasibility of streaming rates. A very intrinsic way of catching this phenomenon, called *power allocation problem*, can be formulated as follows: a given configuration of users which requires some given bit-rates is *feasible* if there exists some vector of emitted powers which guarantee that the Signal-to-Interference-and-Noise-Ratio (SINR) at each receiver exceeds some threshold related to the required bit-rate of the associated channel (given particular coding, modulation and decoding scheme). Even if in practice additional or different feasibility criteria may be implemented in the network controllers (for example based on the maximal emitted power, maximal number of users in each cell, etc.), the above one is universal (does not depend on particular manufacturers' algorithms) and exploits the theoretical limits for the network capacity.

Solving the above power allocation problem is a very difficult, highly centralized problem. So, in this paper we study blocking and cut probabilities for *some particular feasibility criteria* build upon the scalable admission control schemes developed in [1], [2]; they are based on an exact representation of the geometry of interference and ensure that the associated power allocation problems have solutions.

More precisely, we consider a model in which calls arrive to some subset of the plane, representing the multi-cell network, and are subject to some completely independent Markovian mobility during the service. The dynamics of this *free* process is modified each time an arrival or displacement makes the new configuration of users non-feasible (i.e.; not belonging to a given set of feasible configurations). New arrivals in such situations are simply not allowed to enter the system and lost. As far as displacement is concerned, we consider two possible modifications of the free process, which lead to *two different loss models*. The *transition blocking model* (TB), assumes that the moving call in question is immediately taken back to its previous position and the process evolves according to its "free" dynamics. The other one, called *forced termination*

¹ INRIA & Ecole Normale Supérieure and Math. Inst., Univ. of Wrocław, 45 rue d'Ulm, 75005 Paris France, Bartek.Blaszczyszyn@ens.fr,

² France Télécom Research and Development, 38/40 rue du Général Leclerc, 92794 Issy-Moulineaux France, mohamed.karray@orange-ftgroup.com

model (FT), assumes that the moving call is prematurely interrupted (cut).

The models TB and FT are obviously equivalent when there is no user mobility. The TB model is more natural if users backtrack when the quality of their calls becomes unacceptable. It will be mathematically analyzed. The FT model (at a non-null speed) will be simulated only as it is mathematically less tractable. In both models we observe the effect of user mobility on the quality of service.

More precisely, in the TB model, we establish analytical expressions for the (access) blocking probability that under some quite natural assumptions does not depend on the mean user speed. In this model we also analyze the *mean number of blocked motions per call*.

For the FT model, in which, besides call (access) blocking, one observes call cuts, we discover by simulation that the blocking probability decreases, while the cut probability increases when the mean user speed increases. Moreover, the sum of these two probabilities remain roughly constant, and for a moderate speed is well approximated by the blocking probability in the motionless case. We also show that for such a speed the cut probability is well approximated by a normalized version of the mean number of the motion blocking per call evaluated for the corresponding TB model. Finally, combining the above two observations we approximate the blocking probability in the FT model by the difference: of the blocking probability evaluated at the null speed and the normalized mean number of the motion blocking per call. We validate the above approximations studying UMTS release 99.

The remaining part of this paper is organized as follows. In Section II we briefly review the related work and position our contribution in this context. Section III introduces basic notions and tools. In section IV we describe and analyze the TB and FT model. In particular in Section IV-C we introduce our explicit approximations for the blocking and cut probabilities in the FT model. In Section V we develop our main example — analysis of the streaming in the large CDMA networks. We conclude in Section VI. Appendix contains some more technical elements of our mathematical analysis.

II. STATE OF THE ART, RELATED WORK

A large part of the existing literature on performance of cellular networks considers either a random (typically Poisson) snapshot of the population of users ([3], [4]) or some Markovian dynamics of arrivals and departures of motionless users ([5], [6], [7], [8]). In the first case only feasibility probability can be defined, while in the second one blocking probabilities can be studied. In rare articles considering a mobile population of users, where cut probability may be defined, and going further than a pure simulation analysis, the geometry of interference is typically absent or seriously simplified. The most common assumption is that inter-cell interference is proportional to intra-cell interference. This leads to models where only the number of calls per cell (and not their geographic positions) is relevant. Examples of such studies are [9], [10]. In [10] and [11] the cut probability is evaluated considering a “phantom” mobile moving in the network, which does not affect its state.

In [9], which is the most relevant to our approach, two QoS indicators are defined: new call blocking probability and handoff blocking probability. Explicit expressions for these indicators are given for two limiting regimes: no mobility and infinite mobility. Approximations are also given for intermediate mobility regimes. These approximations are based on the study of some special and rather simplistic network architectures (e.g. cells located on a ring or on the line).

In [10] two QoS indicators are defined: blocking probability and forced termination probability. Erlang fix point approximations are used to calculate these probabilities numerically. In [12] upper bounds for the blocking and outage probabilities are derived under a certain monotonicity property.

A. Our contribution

Our model allows to represent in an exact way the geometry of inter-cell and intra-cell interferences. We construct it using the spatial Markov queueing setting ([13]) that, we believe, is more natural in the wireless communication context. At least it allows to simplify many expressions, which in classical Markov approach take complex forms if one wants to discretize reasonably the network geometry.

We identify the feasible configurations of users via the decentralized schemes developed in [1], [2]. This allows to evaluate blocking and cut probabilities for very large (theoretically infinite) cellular networks. This approach is also strongly related to universal (i.e.; not depending on particular manufacturers’ algorithms) theoretical limits for the network capacity based on the feasibility of the power allocation.

In this context, we establish explicit approximations of the blocking and cut probability as functions of the average speed of users. These approximations are shown to be valid up to a reasonable speed.

This paper complements [8], where blocking probabilities without user mobility were studied via a spatial Erlang formula and [14], where the decentralized congestion control schemes were studied in the context of data traffic.

III. PRELIMINARIES

A. Point process description of the system state

Consider a space \mathbb{D} that is always in this paper a subset of the plane \mathbb{R}^2 . Elements $x \in \mathbb{D}$ denote *geographic locations of users* in the system. Configurations $\{x_i\} \subset \mathbb{D}$ of users in the system are identified by corresponding counting measures $\mu = \sum_i \varepsilon_{x_i}$; where the Dirac measure ε_x is defined by $\varepsilon_x(A) = 1$ if $x \in A$ and 0 otherwise, consequently $\mu(A)$ is the number of users in the set $A \subset \mathbb{D}$. We denote by \mathbb{M} the set of *all finite configurations of users* (i.e., finite counting measures) on \mathbb{D} .

B. Spatial Markov queueing process

We will describe the temporal evolution of the configuration of users in \mathbb{D} by a pure jump Markov process, which takes values in \mathbb{M} . A general class of such processes, called *Spatial Markov Queueing* (SMQ) was proposed in [13]. This process evolves because of users arriving, moving or leaving the system, with only one such event being possible at a time. Special cases of SMQ processes are *Spatial Birth-and-Death* and *Markov Poisson Location* (MPL) processes where users arrive, move and leave the system independently of each other.

C. Free (MPL) process

The process describing the evolution of the *mobile streaming users in the absence of any loss-generating constraints* is called *free process*. A natural candidate in the context of wireless communications is the MPL process. Here we describe its dynamics more precisely.

1) *Arrivals, holding times and service rates*: For a given subset $A \subset \mathbb{D}$ inter-arrival times of users to A are independent of everything else, exponential random variables with mean $1/\lambda(A)$, where $\lambda(\cdot)$ is some given *intensity measure of arrivals* to \mathbb{D} per unit of time. In homogeneous traffic conditions, we take $\lambda(dx) = \lambda dx$, where λ is the mean number of arrivals per unit of area and per unit of time. We always assume $\lambda(\mathbb{D}) < \infty$ (thus, in homogeneous case, \mathbb{D} is a finite subset of the plane).

We assume that each arrival stays in the system during some *exponentially distributed service time* (streaming or call holding time during which some given transmission rate should be sustained) with parameter $\tau > 0$. There is no queueing. (The exponential assumption can be relaxed in the subsequent analysis of the transition blocking in the MPL model due to the so-called insensitivity property.)

2) *Mobility*: Assume that users move independently of each other in \mathbb{D} . The *sojourn duration* of a given user at a location $x \in \mathbb{D}$ is independent of everything else (in particular of the amount of the service already received) and *exponentially distributed* (cf [15]) with parameter $\lambda'(x)$. Each user finishing its sojourn at location x , is *routed to a new location* dy according to some *probability kernel* $p'(x, dy)$, where $p'(x, \mathbb{D}) = 1$. The above description corresponds to a Markov process on \mathbb{D} with the following generator (of the individual user mobility): $\lambda(x, dy) = \lambda'(x)p'(x, dy)$. We will always assume that this Markov process is reversible and ergodic and will denote its invariant distribution by $\varrho(\cdot)$; it satisfies the following balance equations: $\varrho(\mathbb{D}) = 1$ and

$$\int_A \lambda(x, \mathbb{D}) \varrho(dx) = \int_{\mathbb{D}} \lambda(x, A) \varrho(dx), A \subset \mathbb{D}. \quad (3.1)$$

3) *Stationary distribution of the free process*: Under the above assumptions the MPL process is reversible and ergodic ([16], [17]). The configuration of users converges weakly to the unique stationary distribution Π , that coincides with the distribution of a Poisson point process on \mathbb{D} of intensity $\rho(\cdot)$ satisfying the following system of traffic equations for $A \subset \mathbb{D}$

$$\lambda(\mathbb{D}) = \tau \rho(\mathbb{D}) \quad (3.2)$$

$$\int_A \lambda(x, \mathbb{D}) \rho(dx) + \tau \rho(A) = \int_{\mathbb{D}} \lambda(x, A) \rho(dx) + \lambda(A).$$

In order to study the general impact of user speed on the performance of the loss system we will multiply the displacement rates $\lambda'(\cdot)$ of all users by a common factor $v \geq 0$ that we will interpret as the *average speed of users*.

Remark 3.1: Note that the solution ϱ of (3.1) is invariant with respect to v . Moreover, if the arrival intensity $\lambda(\cdot)$ is proportional to $\varrho(\cdot)$, i.e.; if $\varrho(\cdot) = \lambda(\cdot)/\lambda(\mathbb{D})$ then the solution ρ of (3.2) is equal to $\rho(\cdot) = \varrho(\cdot)\lambda(\mathbb{D})/\tau$ and thus it is also invariant with respect to v . Consequently, *the stationary distribution Π of the free process with arrival intensity proportional to the invariant distribution of the user mobility does not depend on the average mobility speed v .*

D. Modeling losses

Consider a SMQ process describing a free evolution of the system, e.g. the MPL process described in the previous section. Suppose that the “true” evolution of the system is subject to some constraints, which can be expressed as the limitation of the original state space of all configurations \mathbb{M} of users, to a given fixed subset $\mathbb{M}^f \subset \mathbb{M}$ of *feasible configurations*. We will always assume that \mathbb{M}^f has the following *monotonicity property*: if a configuration μ is feasible ($\mu \in \mathbb{M}^f$) then any subset $\nu \subset \mu$ is also feasible, i.e.; $\nu \in \mathbb{M}^f$.

Examples of \mathbb{M}^f useful in modeling of wireless communication systems are presented in Section V-A2. We remark here only that in general, the feasibility condition (i.e.; the condition for $\mu \in \mathbb{M}^f$) depends not only on the total number of users $\mu(\mathbb{D})$ but also on individual user locations. Note also that the monotonicity property of \mathbb{M}^f implies that all user departures preserve the feasibility of configurations.

We assume that *the “true” system with losses, started at an initial state in \mathbb{M}^f follows the same dynamic as the free process as long as it stays in \mathbb{M}^f and is forced to modify its behaviour each time an attempt of a transition from \mathbb{M}^f to $\mathbb{M} \setminus \mathbb{M}^f$ occurs*. In the next section we will consider two possible modifications applied at such epochs. They lead to two different models, one of which (suitable if users backtrack when the quality of their calls becomes unacceptable) is analytically tractable, the other is more difficult to analyze. Studying both of them we will be able to propose some explicit formulas for evaluation of the loss (blocking and cut) probabilities.

IV. TWO SPATIAL LOSS MODELS

In this section we describe and analyze two models, which consist in two different modifications of the free process dynamics making it stay in the set \mathbb{M}^f of feasible configurations. For simplicity we restrict ourselves to MPL process as the free process. More general scenario is considered in the Appendix. The main results concerning approximations of blocking and cut probabilities are given in Section IV-C.

A. Transition blocking (TB) model

In this model the free process is modified to stay in \mathbb{M}^f by applying transition blocking for the admission as well as for displacements. More precisely, the following rules are applied when an attempt of a transition from \mathbb{M}^f to $\mathbb{M} \setminus \mathbb{M}^f$ occurs.

(TB1): Any call arrival that would result in taking the process to a state outside \mathbb{M}^f is not allowed to enter to the system (lost) and excluded from its further evolution.

(TB2): Any displacements of a user in the system that would take the process to a state outside \mathbb{M}^f is ignored; the user in question is instantaneously taken back to his previous location and the system keeps on evolving with this user according to the free dynamics until the next attempt to leave \mathbb{M}^f .

The above modification of the free process evolution correspond to the so called truncation of its generator (see Appendix).

1) Stationary distribution of the TB process:

Fact 4.1: The TB of the MPL model leads to a reversible and ergodic process, whose stationary distribution Π^{tb} is equal

to the truncation of Π to \mathbb{M}^f ; i.e.; for any subset $\Gamma \subset \mathbb{M}$ of user configurations $\Pi^{\text{tb}}(\Gamma) = \Pi(\Gamma \cap \mathbb{M}^f) / \Pi(\mathbb{M}^f)$.

Proof: The MPL (free) process is reversible with respect to its stationary distribution Π , then the result follows from [13, Proposition 3.14]. ■

2) *Blocking probabilities in the TB model and a spatial Erlang formula:* We define the blocking probabilities in the TB model by the following ergodic limits

$$b^{\text{tb}} = \lim_{t \rightarrow \infty} \frac{\#\{\text{blocked arrivals in } [0, t]\}}{\#\{\text{all arrivals in } [0, t]\}}.$$

where $\#$ denotes the cardinality and $[0, t]$ designates a time interval. One can also consider the blocking probabilities inside the system, i.e.; the ratio the number of blocked displacements with respect to the number of all displacements of users in the system. However this characteristic is not pertinent in our analysis. Instead we define the following *mean number of motion blocking per call*

$$d^{\text{tb}} = \lim_{t \rightarrow \infty} \frac{\#\{\text{blocked displacements in } [0, t]\}}{\#\{\text{non-blocked arrivals in } [0, t]\}}.$$

Both b^{tb} and d^{tb} admit some more explicit expressions in terms of the stationary distribution Π^{tb} . These formulas are derived in the Appendix. In particular, the following formula can be seen as a spatial extension of the well known Erlang formula. It follows from Proposition A.2.

Corollary 4.2: $b^{\text{tb}} = \int_{\mathbb{D}} p^{\text{tb}}(y) \lambda(dy) / \lambda(\mathbb{D})$ where $p^{\text{tb}}(y) = \Pi\{\mu \in \mathbb{M}^f : \mu + \varepsilon_y \notin \mathbb{M}^f\} / \Pi(\mathbb{M}^f)$.

The analogy to the Erlang formula consists in expressing the intensity $p^{\text{tb}}(y)$ of blocking of users arriving at $y \in \mathbb{D}$ by the *conditional probability that the stationary configuration of users in the free (here Poisson) process cannot admit a new user at y given the configuration is in \mathbb{M}^f* . The above result was used in [8] to evaluate the blocking rates in CDMA without user mobility.

Remark 4.3: Note by the above corollary and the Remark 3.1 that if the arrival intensity is proportional to the invariant distribution of the user mobility, then *the blocking probability b^{tb} does not depend on the user speed v* .

The following Corollary follows from Proposition A.3.

Corollary 4.4: $d^{\text{tb}} = \int_{\mathbb{D} \times \mathbb{D}} \Pi\{\mu \in \mathbb{M}^f : \mu + \varepsilon_x \in \mathbb{M}^f, \mu + \varepsilon_y \notin \mathbb{M}^f\} \lambda(x, dy) \rho(dx) / (\Pi(\mathbb{M}^f) \tau \int_{\mathbb{D}} (1 - p^{\text{tb}}(x)) \rho(dx))$.

The proof is given in the Appendix. Note that d^{tb} depends on v even if Π is invariant with respect to v . In fact, in this case d^{tb} increases linearly in v as a consequence of the linear dependence of $\lambda(x, dy)$ on v .

In some cases the value $\Pi\{\dots\}$ in the formula for d^{tb} given in Corollary 4.4 can be evaluated more explicitly.

Corollary 4.5: If $\Pi^{\text{tb}}(N + \delta_x \in \mathbb{M}^f, N + \delta_y \notin \mathbb{M}^f) = \Pi^{\text{tb}}(N + \delta_x \in \mathbb{M}^f) \Pi^{\text{tb}}(N + \delta_y \notin \mathbb{M}^f)$, which is the case e.g. when \mathbb{M}^f is in the form (5.3) then $d^{\text{tb}} = \int_{\mathbb{D} \times \mathbb{D}} p^{\text{tb}}(y) (1 - p^{\text{tb}}(x)) \lambda(x, dy) \rho(dx) / \int_{\mathbb{D}} \tau (1 - p^{\text{tb}}(x)) \rho(dx)$.

B. Forced termination (FT) model

In this model the dynamics of the free process is modified according to the following rules when an attempt of a transition from \mathbb{M}^f to $\mathbb{M} \setminus \mathbb{M}^f$ occurs.

(FT1): Any call arrival that would result in taking the process to a state outside \mathbb{M}^f is not allowed to enter to the system and excluded (lost) from its further evolution.

(FT2): Any displacements of a user in the system that would take the process to a state outside \mathbb{M}^f leads to the forced termination (cut) of the call of this user, i.e.; rejection of this user from the system and from its further evolution.

Each time the rule (FT1) or (FT2) is applied we say that the corresponding user (call) is lost.

1) *Blocking and cut probabilities in FT model:* The FT of the MPL model is ergodic (this and all the statements on the FT models are proved in [17], see also [16]). Let's denote by Π^{ft} the stationary distribution of the FT process; it can be characterized by the so called cycle formula. The main disadvantage of the FT model is that Π^{ft} cannot be expressed more explicitly even in the simplest examples.

Our main performance metrics of the FT model are blocking and cut probabilities defined respectively by the following ergodic limits

$$b^{\text{ft}} = \lim_{t \rightarrow \infty} \frac{\#\{\text{blocked arrivals in } [0, t]\}}{\#\{\text{all arrivals in } [0, t]\}},$$

$$c^{\text{ft}} = \lim_{t \rightarrow \infty} \frac{\#\{\text{forced call terminations in } [0, t]\}}{\#\{\text{non-blocked arrivals in } [0, t]\}},$$

which exist and can be expressed in terms of the stationary distribution Π^{ft} . However, since this distribution is typically not known explicitly, the FT blocking and cut probabilities are studied by simulation. Examples of numerical results are presented in Section V-C. The following conclusion can be derived from these simulations.

Observation 4.6: In the FT model with \mathbb{M}^f of the form (5.2) or (5.3) the blocking probability decreases as a function of user mobility speed v , while the FT cut probability increases in v , in such a way that their sum remains roughly constant, at least for small and moderate v .

C. Approximations

In this section we propose some approximations of the blocking and cut probabilities in the FT model, which can be calculated using the TB model. These approximations are heuristic, but useful, as shown in Section V where the numerical results are presented.

1) *Approximating cut probabilities:* Our approximation for the cut probability is based on the mean number of motion blocking per call d^{tb} and the following intuitive reasoning: *d^{tb} should be close to c^{ft} when this latter is small, while it tends to infinity when c^{ft} tends to 1*. Bearing in mind the above idea we propose to approximate c^{ft} by the following *normalized number of motion blocking per call*

$$c^{\text{ft}} \approx c^{\text{tb}} := \frac{d^{\text{tb}}}{1 + d^{\text{tb}}}. \quad (4.3)$$

The quantity c^{ft} can be seen also as a “fictitious” cut probability defined in the TB model (note that in this model there are no “real” call cuts).

2) *Approximating blocking probabilities:* Our approximation for the blocking probability is based on Observation 4.6, which says that $b^{\text{ft}} + c^{\text{ft}}$ is approximately independent of mobility speed v at least for small and moderate values of v . Considering the null speed, one concludes that $b^{\text{ft}} + c^{\text{ft}} \approx b_0$, where this latter denotes the (access) blocking probability evaluated for $v = 0$ (equivalently in TB or FT model). Bearing

on mind the above reasoning and the approximation (4.3) we propose to approximate the blocking probability in the FT model by the following formula

$$b^{\text{ft}} \approx b_0 - c^{\text{tb}}. \quad (4.4)$$

V. STREAMING IN CDMA

In this section we will show how the ideas developed in the previous section can be used to analyze the performance of the UMTS release 99 cellular network. We consider a large multi-cellular network serving streaming users, which are subject to inter- and extra-cell (handoffs) mobility.

A. Network topology and feasible configurations of users

1) *Hexagonal cell pattern*: We consider the most popular hexagonal model, where the base stations are placed on a regular hexagonal grid. Let R be the radius of the disc whose area is equal to that of the hexagonal cell served by each base station, and call R the *cell radius*. Bearing in mind the above pattern of base stations, one can consider $\mathbb{D} = \bigcup_u C_u$, where $C_u \subset \mathbb{R}^2$ are hexagonal cells constituting the network.

2) *Feasible configurations*: It is natural to identify the feasible configurations of users in the network \mathbb{D} studying the feasibility of power allocation problem (cf. Introduction). In this approach, a given configuration of users with predefined bit-rates is *feasible* if there exists some vector of emitted powers which guarantee that the Signal-to-Interference-and-Noise-Ratio (SINR) at each receiver exceeds some threshold related to the bit-rate of the associated channel. However, solving the power allocation problem for a large network is a very complicated task.

a) *Sufficient condition*: A sufficient condition for the feasibility of power allocation is derived in [1], [2]. The verification of this condition is decentralized in the sense that it can be implemented in such a way that, for a given architecture of the network, each base station u only has to check whether the following inequality holds for the users of its own cell

$$\int_{C_u} \varphi_u(x) \mu(dx) = \sum_{x_i \in \mu \cap C_u} \varphi_u(x_i) < C_u, \quad (5.1)$$

where C_u is some constant, $\varphi_u(\cdot)$ is some nonnegative function of the user $x \in C_u$ location and the service rate (see [17, Prop. 57, p. 226]) and the integral with respect to the counting measure $\mu(dx)$ in C_u denotes summation over all users x_i in μ present in cell C_u . The above approach suggests the following conservative choice for the set of feasible configurations \mathbb{M}^f of users in the *geometric model* $\mathbb{D} = \bigcup_u C_u$ of the network

$$\mathbb{M}^f = \{\mu \in \mathbb{M} : \int_{C_u} \varphi_u(x) \mu(dx) < C_u \forall \text{cell } u\}. \quad (5.2)$$

b) *Erlang-type condition*: Recall that in the steady state of MPL model users are distributed according to a Poisson point process (see Section III-C3). This means that the number of terms $\mu(C_u)$ in the sum/integral in (5.1) and (5.2) has a Poisson distribution, and that given this number, the individual terms $\varphi_u(x_i)$ are i.i.d. random variables. In this case, discretizing C_u (e.g. partitioning it into several rings around the base station) and applying the linear regression of the sum $\sum_{x_i \in \mu \cap C_u} \varphi_u(x_i)$ versus the number of users in each element of the partition one can reduce the geometric model to some multi-rate model, in which the blocking probability can be evaluated e.g. via the Kauffman-Roberts algorithm ([18]). In

the most simple case, when C_u is not partitioned at all, the linear regression with respect to $\mu(C_u)$ leads to the following Erlang-type approximation of \mathbb{M}^f

$$\mathbb{M}^f = \{\mu \in \mathbb{M} : \mu(C_u) \leq C/\bar{\varphi}_u \forall \text{cell } u\}, \quad (5.3)$$

where $\bar{\varphi}_u = \int_{C_u} \varphi_u(x) \rho(dx) / \rho(C_u)$ denotes the mean value of $\varphi_u(x)$. In this case the geometric model boils down to a discrete one.

c) *Other conditions*: The above models of \mathbb{M}^f are based on the theoretical analysis of the power allocation problem. For some particular network controllers one can also consider really implemented admission and congestion control algorithms and derive corresponding feasible user locations. Most of such algorithms are based on the total emitted power which is required to be at most half of its maximal value.

Note also that our theoretical analysis is derived for the network equipped with matched filter receivers. More advanced reception techniques (e.g. successive interference cancellation) will lead to different (less constraining) sets \mathbb{M}^f .

3) *Decentralizing large network*: It can be shown in the hexagonal network that functions $\varphi_u(\cdot)$ do not depend on u provided cell u is not on the border of the network (precisely, the value of the interference factor f comprised in $\varphi_u(\cdot)$ given in [17, p. 217] is practically determined by 4 rings of neighbouring cells). It is reasonable to assume similar property for the mobility model $\lambda(x, dy)$. Studying the performance of a large network, one can ignore the border effects and consider the network that is “wrapped around”; i.e., deployed on a torus.

Moreover, in the case of \mathbb{M}^f given by (5.2) or (5.3) blocking rates $p^{\text{tb}}(y)$ can be evaluated studying only the cell to which belongs y . Similarly, Corollary 4.5 shows that for a symmetric network (as a toroidal one) and the mobility model that allows only for local handoffs, mean number of blocked motions per call d^{tb} can be mathematically evaluated studying a single (call it “typical”) cell of the network with its direct neighbours. Similar conclusion can be drawn for a large “flat” (not toroidal) network using spatial ergodic arguments.

B. Model specification

1) *Network architecture*: We consider a cell radius $R = 1\text{km}$ and take $4 \times 4 = 16$ toroidal cell model.

2) *Arrivals and call duration*: We take mean call duration $1/\tau = 2\text{min}$. We assume the (spatially) uniform arrival stream $\lambda(dx) = \lambda dx$ with λ varying such that the *traffic demand* $\lambda \pi R^2 / \tau$ varies from 0 to 120 Erlangs per cell.

3) *User mobility*: We assume a simple *completely aimless* Markovian mobility model ([19], [20]) that yields the uniform stationary distribution ϱ of user location in the network. In view of the above arrival/call duration specification it implies also the homogeneous solution of the traffic equations (3.2) of the form $\rho(dx) = \lambda(dx)/\tau$ (see [17] for more details). We will consider three mobility regimes with *mean user speed* $v = 0.1, 1, 10 \text{ km per mean call duration}$, which correspond, respectively, to 3, 30 and 300 km/h when the mean call duration is $1/\tau = 2\text{min}$.

4) *Other parameters*: We assume a path loss $L(r) = (Kr)^\eta$, with $\eta = 3.38$, $K = 8667$. We consider voice calls with SINR threshold -16dB . We take an orthogonality factor (for intracell interference) 0.4, maximal power 52dBm and the ambient noise power -103dBm .

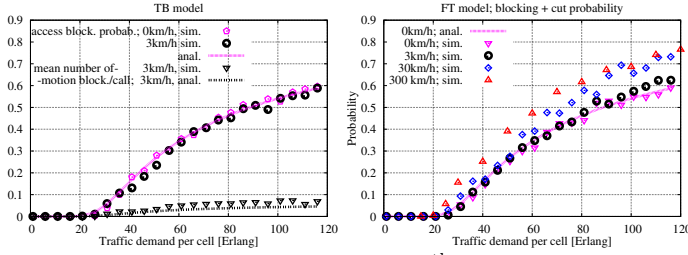


Fig. 1. Left: Study of the blocking probability b^{tb} and the mean number of motion blocking per call d^{tb} in the TB model. Right: sum of the blocking and cut probability simulated in the FT model.

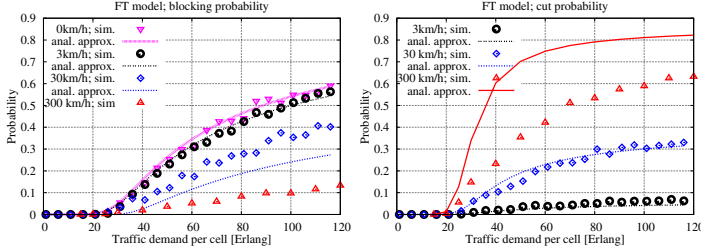


Fig. 2. Study of the FT model.

C. Numerical results

1) *Study of the TB model:* Figure 1 (left) shows the blocking probability b^{tb} and the mean number of motion blocking per call d^{tb} in the TB model under null and a pedestrian user speed (when it is reasonable to assume that users backtrack). The analytic curves are obtained from our explicit formulas. As expected in this model, the blocking probability does not depend on the speed.

2) *Study of the FT model:* Note first on the Figure 1 (right) that the sum of the blocking and cut probability in the FT model is nearly independent of the speed. Next, simulated data on Figures 2 (left and right) show that blocking probability decreases while the cut probability increases when the mean user speed increases. These figures show also heuristic approximations of these performance metrics of the FT model described in Section IV-C and obtained via the corresponding TB model. These approximations seem to be accurate up to a moderate vehicular user speed of 30km/h or when the blocking/cut probability is less than 0.05 that is a reasonable threshold for real networks.

VI. CONCLUDING REMARKS

In this paper we have proposed two models allowing to study the impact of user mobility on the performance of cellular networks serving streaming traffic. In the model without backtrack (with call cuts) we have observed that the blocking probability decreases as a function of user mobility speed, while the cut probability increases, in such a way that their sum remains constant for small and moderate mobility speed. In the model with backtrack we have explicitly expressed the blocking probability and the mean number of motion blocking per call. We have also shown how the performance metrics of the former model can be explicitly approximated via the analysis of the later one. We have validated this approach studying the UMTS release 99 cellular network serving voice traffic under completely aimless mobility model. Similar approach can be used to study OFDMA networks. An

interesting open question is the impact of the particular *form* of the mobility model on blocking and cut probabilities.

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APPENDIX: MATHEMATICAL BACKGROUND

In this section we prove a few results for the TB model, which are used in the paper. More on the TB and FT model can be found in [16], [17].

As usually one introduces a “virtual” state $o \notin \mathbb{D}$ which can be seen as a location outside the space \mathbb{D} , from which users arrive to the system and which represents the destination of the users leaving the system. Denote $\bar{\mathbb{D}} = \mathbb{D} \cup \{o\}$ and \mathbb{M} the set of finite counting measures on \mathbb{D} . Define the following *displacement operator* T on the space \mathbb{M} for $\nu \in \mathbb{M}, x \in \nu, y \in \mathbb{D}$: $T_{oy}\nu = \nu + \varepsilon_y$, $T_{xo}\nu = \nu - \varepsilon_x$, and $T_{xy}\nu = \nu - \varepsilon_x + \varepsilon_y$. It is customary to define also $T_{AB}\nu = \{T_{xy}\nu : x \in A, y \in B, x \neq y\}$ for $A, B \subset \bar{\mathbb{D}}, \nu \in \mathbb{M}$.

Consider a SMQ generator q ([17]) representing a free process, with the *routing kernel* $\lambda(x, A)$ ($x \in \bar{\mathbb{D}}, A \in \bar{\mathcal{D}}$) (with $0 \leq \lambda(x, \bar{\mathbb{D}}) < \infty$ and $\lambda(x, \{x\}) = 0$ for all $x \in \bar{\mathbb{D}}$) and the *departure-arrival rate* $r(\nu, T_{xy}\nu)$ ($x, y \in \bar{\mathbb{D}}, \nu \in \mathbb{M}$) for the displacement from x to y . In what follows we always assume that $0 \leq r(\nu, T_{xy}\nu) < \infty$ for all $\nu \in \mathbb{M}, x, y \in \bar{\mathbb{D}}$ and $x \in \nu$ or $x = o$.

Consider a fixed (measurable) subset $\mathbb{M}^f \subset \mathbb{M}$ of feasible states. We assume that if $\mu \in \mathbb{M}^f$ then for any $\nu \subset \mu$ one has $\nu \in \mathbb{M}^f$ (we called this *monotonicity property* of \mathbb{M}^f).

The dynamics of the TB process (cf Section IV-A) related to the free process given by the generator q corresponds to the following truncation of q : $q^{tb}(\nu, \Gamma) = q(\nu, \Gamma \cap \mathbb{M}^f)$ if $\nu \in \mathbb{M}^f$ and $q(\nu, \Gamma)$ otherwise. One can show (see [17]) that q^{tb} is also a SMQ generator, with the same routing kernel λ and the departure-arrival rates $r^{tb}(\nu, T_{xy}\nu) = r(\nu, T_{xy}\nu)1(T_{xy}\nu \in \mathbb{M}^f)$ if $\nu \in \mathbb{M}^f$ and $r(\nu, T_{xy}\nu)$ otherwise.

Suppose that q is a regular, ergodic SMQ generator and denote its stationary distribution by Π . Consider the TB process $\{N_t\}$ given by the above truncation of the generator q . In what follows we assume that the TB process is also ergodic and has a particular form of the limiting distribution $\Pi^{tb}(\cdot) = \Pi(\cdot \cap \mathbb{M}^f) / \Pi(\mathbb{M}^f)$ being the *truncation* of Π to \mathbb{M}^f . This truncation property does not always hold, and one simple sufficient condition for this to hold is when the original free process given by q is reversible (cf [13, Proposition 3.14]).

In order to formalize the notion of the blocking probability and blocked displacements one models the time-epochs and departure-arrival locations of these blocked transitions by a double stochastic Poisson point process $\Phi_0 = \sum_i \delta_{(t_i, x_i, y_i)}$ driven by TB process $\{N_t\}$, where t_i, x_i, y_i denote, respectively, the time-epochs, departure and arrival locations of blocked transitions. Given a realization $\{N_t\}$ of the TB process, Φ_0 is a Poisson point process with intensity measure Λ_{N_t} on $(0, \infty) \times (\bar{\mathbb{D}})^2$, given by $\Lambda_{N_t}(D \times A \times B) = \int_D q(N_t, T_{AB}N_t \setminus \mathbb{M}^f) dt$. Denote also by Φ_1 the point process on $(0, \infty) \times (\bar{\mathbb{D}})^2$ associated to (“true”) transitions of N_t ; i.e., $\Phi_1(D \times A \times B) = \sum_{s>0} 1(s \in D, N_s = T_{xy}N_{s-}, x \in$

$A, y \in B$). Let $\Phi = \Phi_0 + \Phi_1$ be the superposition of Φ_i , $i = 0, 1$. Finally define the blocking probability for the transitions $\nu \rightarrow T_{AB}(\nu)$ for some $A, B \in \mathbb{D}$ and $\nu \in \mathbb{M}^f$ (we will call them transitions from A to B for short) as the following limiting ratio of blocked transitions to all transitions

$$p_{AB}^{\text{tb}} = \lim_{t \rightarrow \infty} \frac{\Phi_0((0, t] \times A \times B)}{\Phi((0, t] \times A \times B)} \quad (\text{A.1})$$

The above limit exists by the following result.

Lemma A.1: Suppose that \emptyset is a positive recurrent state for q^{tb} (which is true in particular if q^{tb} is ergodic) with the limiting distribution Π^{tb} . If

$$\mathbf{E}_{\Pi^{\text{tb}}}[q(N, \mathbb{M})] < \infty \quad (\text{A.2})$$

then $\lim_{t \rightarrow \infty} \frac{1}{t} \Phi_0((0, t] \times A \times B) = \mathbf{E}_{\Pi^{\text{tb}}}[q(N, T_{AB}N \setminus \mathbb{M}^f)]$ and $\lim_{t \rightarrow \infty} \frac{1}{t} \Phi_1((0, t] \times A \times B) = \mathbf{E}_{\Pi^{\text{tb}}}[q(N, T_{AB}N \cap \mathbb{M}^f)]$ a.s. for any initial value $N_0 = \nu$ of the TB process, for which the return time to \emptyset is a.s. finite.

Proof: Consider a probability space on which the TB process $\{N_t\}_t$ and both point processes Φ_i ($i = 0, 1$) are (time) stationary. Denote by $\mathbf{E}_{\Pi^{\text{tb}}}$ the expectation corresponding to the stationary distribution of $\{N_t\}_{t \geq 0}$. Condition (A.2) implies that the point process Φ_1 has finite intensity. Indeed, $\mathbf{E}_{\Pi^{\text{tb}}}[\Phi_1((0, 1] \times \mathbb{D} \times \mathbb{D})] = \mathbf{E}_{\Pi^{\text{tb}}}[q^{\text{tb}}(N_0)] \leq \mathbf{E}_{\Pi^{\text{tb}}}[q(N_0, \mathbb{M})] < \infty$ where the equality follows from the Lévy's formula. Similarly, the intensity of Φ_0 that is a doubly stochastic Poisson point process is finite $\mathbf{E}_{\Pi^{\text{tb}}}[\Phi_0((0, 1] \times \mathbb{D} \times \mathbb{D})] = \int_0^1 \mathbf{E}_{\Pi^{\text{tb}}}[q(N_t, \mathbb{M} \setminus \mathbb{M}^f)] dt \leq \mathbf{E}_{\Pi^{\text{tb}}}[q(N_0, \mathbb{M})] < \infty$. For given $A, B \subset \mathbb{D}$ the processes $X_t^i = \Phi_i((0, t] \times A \times B)$ ($i = 1, 2$) are cumulative with the imbedded renewal process being the epochs of successive visits of N_t at \emptyset . Thus $\lim_{t \rightarrow \infty} \frac{1}{t} \Phi_1((0, t] \times A \times B) = \mathbf{E}_{\Pi^{\text{tb}}}[\Phi_1((0, 1] \times A \times B)] = \mathbf{E}_{\Pi^{\text{tb}}}[q(N_0, T_{AB}N_0 \cap \mathbb{M}^f)]$, where the second equality follows from Lévy's formula. Similarly, by the fact that Φ_0 is a doubly stochastic Poisson point process $\lim_{t \rightarrow \infty} \frac{1}{t} \Phi_0((0, t] \times A \times B) = \mathbf{E}_{\Pi^{\text{tb}}}[\Phi_0((0, 1] \times A \times B)] = \mathbf{E}_{\Pi^{\text{tb}}}[\Lambda_N((0, 1] \times A \times B)] = \mathbf{E}_{\Pi^{\text{tb}}}[q(N_0, T_{AB}N_0 \setminus \mathbb{M}^f)]$. This completes the proof. ■

The following result immediately follows from Lemma A.1.

Proposition A.2: If the conditions of Lemma A.1 are satisfied, then

$$p_{AB}^{\text{tb}} = \mathbf{E}_{\Pi^{\text{tb}}}[q(N, T_{AB}N \setminus \mathbb{M}^f)] / \mathbf{E}_{\Pi^{\text{tb}}}[q(N, T_{AB}N)].$$

The number of blocked displacement per user introduced in Section IV-A can be formally defined as follows

$$d^{\text{tb}} = \lim_{t \rightarrow \infty} \frac{\Phi_0((0, t] \times \mathbb{D} \times \mathbb{D})}{\Phi((0, t] \times \mathbb{D} \times \{o\})} \quad (\text{A.3})$$

The following result follows from Lemma A.1.

Proposition A.3: If the conditions of Lemma A.1 are satisfied then

$$d^{\text{tb}} = \mathbf{E}_{\Pi^{\text{tb}}}[q(N, T_{\mathbb{D}\mathbb{D}}N \setminus \mathbb{M}^f)] / \mathbf{E}_{\Pi^{\text{tb}}}[q(N, T_{\mathbb{D}o}N)].$$

We will give now the *proof of Corollary 4.4*.

Proof: We use Proposition A.3. Recall that in the case of the MPL free process Π is the distribution of the Poisson point process with intensity ρ , and Π^{tb} is the truncation of Π to \mathbb{M}^f . Thus, the denominator in the formula for d^{tb} given in

Proposition A.3 is equal to

$$\begin{aligned} \mathbf{E}_{\Pi^{\text{tb}}}[q(N, T_{\mathbb{D}o}N)] &= \tau \mathbf{E}_{\Pi^{\text{tb}}}[N(\mathbb{D})] \\ &= (\Pi(\mathbb{M}^f))^{-1} \tau \mathbf{E}_{\Pi} \left[\int_{\mathbb{D}} 1(N \in \mathbb{M}^f) N(dx) \right] \\ &= (\Pi(\mathbb{M}^f))^{-1} \tau \int_{\mathbb{D}} \mathbf{E}_{\Pi}[1(N + \varepsilon_x \in \mathbb{M}^f)] \rho(dx) \\ &= \tau \int_{\mathbb{D}} (1 - p^{\text{tb}}(x)) \rho(dx), \end{aligned}$$

where the last but one equality follows from the Campbell formula. Similarly one can show using Campbell formula that $\mathbf{E}_{\Pi^{\text{tb}}}[1(T_{xy}N \notin \mathbb{M}^f)N(dx)] = \Pi(\mathbb{M}^f)^{-1} \Pi(N + \delta_x \in \mathbb{M}^f, N + \delta_y \notin \mathbb{M}^f) \rho(dx)$. Applying the above expression to the numerator in the formula for d^{tb} given in Proposition A.3 one concludes the proof. ■

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