Linear-Regression Estimation of the Propagation-Loss Parameters Using Mobiles' Measurements in Wireless Cellular Networks

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Abstract

We propose a new linear-regression model for the estimation of the path-loss exponent and the parameters of the shadowing from the propagation-loss data collected by the mobiles with respect to their serving base stations. The difficulty consists in deriving the parameters of the distribution of the propagation loss with respect to an arbitrary base station from these regarding the strongest one. The proposed solution is based on a simple, explicit relation between the two distributions in the case of infinite Poisson network and on the convergence of an arbitrary regular (in particular hexagonal) network to the Poisson one with increasing variance of the shadowing. The new approach complements existing methods, in particular the one based on COST Walfisch-Ikegami model, which does not allow for the shadowing estimation and is not suited for indoor scenario.

Index Terms

Path-loss exponent, log-normal shadowing, cellular network, outdoor/indoor, linear regression, hexagonal, Poisson, comparison.

I. Introduction

Modeling of the propagation loss is an important element of the design and performance analysis of wireless systems. This propagation loss results from various physical mechanisms such as reflection, diffraction, scattering and multi-path interference; [1, §3.4]. In the statistical approach, the propagation loss is typically modeled by three factors: *deterministic function of the distance*, which represents average path-loss on the given distance in the network, and two random variables, called *shadowing* and *fading* ([2]) normalized to have mean one, that take into account in a statistical manner the deviation from this average, observed for each particular pair of emitter and receiver. The deterministic path loss function is commonly assumed to be of the form $(Kr)^\beta$ where K, β are constants, with β called *path-loss exponent*. Shadowing S, which takes into account reflection, diffraction, and scattering, is often assumed to have *log-normal distribution*, with $\mathbf{E}[S] = 1$, and parametrized by its variance or, more often, the standard deviation $\sigma_{\mathrm{dB}(S)}$ of S expressed in dB (sometimes called *logarithmic standard deviation*). Fading, that

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takes into account additional small-distance-(and-time)-scale variability due to multi-path interference will not be considered in this paper (hence we are interested in the propagation-loss averaged over this small-scale variability).

The parameters of this simple model need to be specified for every given network scenario. The path-loss exponent β and the constant K for outdoor communications (when mobile is outside a building) are usually taken from generic models (like Okumura-Hata's or Walfisch-Ikegami). There is no well established analogous models for indoor communications (mobile inside a building). Regarding the shadowing, $\sigma_{\mathrm{dB}(S)}$ is usually estimated from exhaustive, time consuming and costly real-network measurements.

In this paper we propose a novel approach consisting in estimation of the path-loss exponent β and some function of K and $\sigma_{\mathrm{dB}(S)}$ of the shadowing (precisely the value of $\tilde{K} = K/\sqrt{\mathbf{E}[S^{2/\beta}]}$) directly from the measurements regarding the propagation loss between base stations (BS) and their served mobiles, usually collected by the operators.

The method is valid regardless of the (indoor or outdoor) scenario. For outdoor scenario, our model can be validated (regarding the path-loss exponent β) and combined (to fix the normalizing constant K) with Okumura-Hata's or Walfisch-Ikegami model, thus allowing to estimate $\sigma_{\mathrm{dB}(S)}$ of the outdoor shadowing without additional real-network measurements.

Even if one does not have any additional information to estimate separately K and $\sigma_{\mathrm{dB}(S)}$, knowing the value of $\tilde{K}$ is enough to express the distribution of many quantities of interest related to propagation, like path-loss with respect to the serving BS, or the interference factor.

Our approach is based on the following two facts:

- A sufficiently large Hexagonal ("regular") network of BS, subject to sufficiently large variance of the shadowing, is perceived at a given location statistically in the same manner (we make this precise using Kolmogorov-Smirnov tests) as some equivalent infinite Poisson (equivalent "irregular") network of BS without shadowing. This means that the joint distribution of the propagation-losses from the strongest BS, second-strongest, and so on, tend to be equal.
- For the infinite Poisson network of bases stations, there exists a simple analytical expression for the cumulative distribution function (cdf) of the propagation loss between a given location and the corresponding strongest BS.

Fitting the expression for the cdf in the equivalent "irregular" (Poisson) network to the data collected by mobiles, allows to estimate the path-loss exponent β and the value of $\tilde{K}$. Moreover, on a doubly logarithmic scale, this fitting boils down to a simple linear regression problem, with $2/\beta$ exponent being the multiplicative factor.

A. Related works

There are several models for propagation loss in outdoor. Deterministic ("exact") techniques, such as the ray tracing [3–6] are time consuming and require a precise relief and building data basis. On the other hand, statistical techniques as Okumura-Hata's model [7, 8] (usually complemented by the log-normal shadowing) do not require such relief information, and give the loss due to distance with a few number of parameters. Other examples are Spatial Channel Model [9] proposed by the 3GPP and ITU-Advanced Channel Model [10]. An intermediate approach consists in providing parameters describing antennas, buildings, streets, etc, which allow to construct first statistical models for the relief, from which macroscopic propagation loss is derived next. This approach is adopted in COST Walfisch-Ikegami model [11, §4.4.1].

Taking into account indoor loss is more tricky. Some studies can be found in [12, 13] that account for the indoor effect through an additional log-normal random variable, or [14–16], which take into account building parameters, and finally [3], which extends the ray-tracing method.

II. Model Description and Methodology

A. Propagation in Network with Shadowing

Denote by $\Phi = \{X_i\}$, $X_i \in \mathbb{R}^2$, a finite or infinite set of locations of base stations in the network. For a given BS $X \in \Phi$ and a given location $y \in \mathbb{R}^2$ on the plane we denote by $L_X(y)$ the (time-average, i.e., averaged out over the fading) propagation-loss between BS X and location y. We assume that $L_X(y) = (|K(X-y)|)^\beta / S_X(y)$, where $K > 0$ and $\beta > 2$ are some constants and $S_X(y)$ is a log-normal random variable of shadowing with mean 1, for every $X \in \Phi$, $y \in \mathbb{R}^2$. Recall that such a mean-1 log-normal variable S can be expressed as $S = e^{\mu+\sigma N}$ where N is the standard Gaussian random variable (with mean 0 and variance 1) with $\mu = -\sigma^2/2$. Note that path-loss expressed in dB, i.e., $\mathrm{dB}(L_X(y))$, where $\mathrm{dB}\,(x) := 10 \times \log_{10}(x)\,\mathrm{dB}$, is a Gaussian random variable with standard deviation $\sigma_{\mathrm{dB}(S)} = \sigma 10/\log 10 \mathrm{dB}$.[1]

In what follows we will always assume that the random fields $\{S_{X_i}(\cdot)\}$ are *independent across BSs* X_i. Our model does not require any particular assumption of the dependence of $\{S_{X_i}(y)\}$ across y for fixed X_i (since the whole analysis of this paper will regard a given fixed location y).

B. Serving Base Station

We assume that a given location $y \in \mathbb{R}^2$ is served by the BS $X^*_y \in \Phi$ with respect to which it has the weakest path-loss $L_{X^*_y}(y)$, i.e, such that $L_{X^*_y}(y) \le L_X(y)$ for all $X \in \Phi$, with any tie-breaking rule.

[1] The assumption $\mathbf{E}[S] = 1$ is a matter of convention. This convention is adopted, e.g., in the COST Walfisch-Ikegami model, cf. [11, §2.1.6 and §4.4.1]. Another option, taken e.g. in [2] is to assume $\mathbf{E}[\mathrm{dB}(S)] = 0\mathrm{dB}$ which is equivalent to our model with the constant K replaced by $e^{\sigma/(2\beta)}K$.

By the definition $L^*(y) := L_{X^*_y}(y)$ is the path-loss experienced by a user located at y with respect to its serving BS.

The distribution of $L^*(y)$, for a "typical" location y in the network, can be estimated form path-loss data reported by mobiles to their serving BS, and usually collected by the operators. The problem is that no analytical form of this distribution is known for a usual (say regular hexagonal) architecture Φ of BS. Hence there is no expression to fit the empirical data for the estimation of parameters. To cope with this deficiency, we use an infinite Poisson architecture which allows for an explicit expression of the cdf of $L^*(y)$, as shown in the next section. The reason for which we can do this is that any large, regular (as e.g. hexagonal) network, with sufficiently large variance of the shadowing can be approximated by an "equivalent" Poisson network, as explained in Section II-E.

C. Poisson Network Architecture

In this section we assume that the BS locations Φ are modeled by a homogeneous Poisson point process of intensity λ (BS per unit of surface). We will give an expression for the cdf of the propagation loss from the serving BS $L^* = L^*(0)$, where by the stationarity we have assumed (without loss of generality) that the mobile is located at the origin $y = 0$.

Fact 1: For Poisson network with intensity λ and arbitrary distribution of S *we have* $\Pr(L^* \geq t) = \exp\left[-\lambda\pi \mathbf{E}\left[S^{2/\beta}\right] t^{2/\beta}/K^2\right]$, *provided* $\mathbf{E}[S^{2/\beta}] < \infty$.

Proof: The expression can be derived from [17, Prop. 5.5] and the general formula for the distribution of the extremal shot-noise; cf. [18, Prop. 2.13]. [2] The key steps are as follows:

$$\Pr(L^* \geq t) = \mathbf{E}\left[\exp\Big[\sum_{X_i\in\Phi} \log \mathbb{1}(L_{X_i}(0) \geq t)\Big]\right] = \exp\left[-\lambda \int_{\mathbb{R}_+}\int_{\mathbb{R}^2} \mathbb{1}((K|x|)^\beta/s < t)\mathrm{d}x F_S(\mathrm{d}s)\right],$$

where $F_S(\cdot)$ is a general cdf of S. Passing to polar coordinates, and straightforward evaluation of the integrals complements the proof. ■

For the normalized ($\mathbf{E}[S] = 1$) log-normal variable S we have $\mathbf{E}\left[S^{2/\beta}\right] = \exp[\sigma^2(2-\beta)/\beta^2]$.

Remark 2: The result regarding the insensitivity of the distribution of L^* in the infinite Poisson model, with respect to the distribution of S, given the $\mathbf{E}[S^{2/\beta}]$, can be generalized to *any network characteristic* that is entirely defined by the collection of propagation losses $\{L_{X_i}(0) = (K|X_i|)^\beta/S_{X_i}(0) : X_i \in \Phi\}$ experienced by the mobile at the given location (as, e.g., the value of the signal-to-interference ratio with respect to the serving station); cf. [17, the proof of Prop. 5.5]. This means that *the infinite Poisson network with an arbitrary shadowing S is perceived at a given location statistically in the same manner as an "equivalent" infinite Poisson with "constant shadowing"* equal to $s_{const} = (\mathbf{E}[S^{2/\beta}])^{\beta/2}$ (to have

[2] It can be aslo found in [19, Corollary 7.4.2], or [20, §3.1]).

the same moment of order $2/\beta$). The model with such a "constant shadowing" boils down to the model without shadowing ($S \equiv 1$) and the constant K replaced by $\tilde{K} = K/\sqrt{\mathbf{E}[S^{2/\beta}]}$.

D. Linear-regression estimation of parameters

We will now show how to estimate the propagation characteristics from measurements of the distribution of the propagation loss with the best server using Poisson model. We deduce from Fact 1 that

$$\log\left(-\log\left(\Pr\left(L^* \geq \log t\right)\right)\right) = \log\left(\frac{\lambda\pi\mathbf{E}\left[S^{2/\beta}\right]}{K^2}\right) + \frac{2}{\beta}t = a + bt\,, \tag{1}$$

where $b = \frac{2}{\beta}$ and $a = \log\left(\lambda\pi\mathbf{E}\left[S^{2/\beta}\right]/K^2\right) = \log(\lambda\pi/\tilde{K}^2)$. Consequently, if the cdf of L^* is available from measurements (or simulations), then we may get a and b by a linear regression between $\log\left(-\log\left(\Pr\left(L \geq t\right)\right)\right)$ and $\log t$. This characterizes the path-loss exponent β and the constant $\tilde{K}$ to be used with the "equivalent" infinite Poisson model without shadowing; cf. Remark 2. (Note that the mean number of base-stations λ per unit of surface is known by the operator.)

Regarding the original model with the normalized log-normal shadowing, the above expression for $\tilde{K}$ gives the following equation involving the parameters K and σ.

$$Ke^{\sigma^2\frac{\beta-2}{2\beta^2}} = \tilde{K}\,. \tag{2}$$

Remark 3: In the case of outdoor measurements, the above linear-regression estimation approach can be validated and completed by other existing propagation models, for example the COST Walfisch-Ikegami propagation model [11]. The validation consists in comparison of the obtained values of β. Completion consists in using the value of the constant K given by this complementary method to deduce from (2) the unknown value of σ (characterizing the shadowing ignored by COST Walfisch-Ikegami model). Following this approach we obtain

$$\sigma_{\mathrm{dB}(S)} = \frac{10}{\log 10}\left(\frac{2\beta^2}{\beta-2}\log\left(\frac{\tilde{K}}{K}\right)\right)^{1/2}\mathrm{dB}\,. \tag{3}$$

E. Approximating Regular Networks with Log-Normal Shadowing by Poisson Ones

Having seen how to use Poisson model to estimate the parameters of the propagation, in this section we will show why and when one can use this model in the context of a "regular" network. Due to space constraint, we will consider only hexagonal network architectures and we will only statistically compare the empirical (simulated) distribution of L^* for such networks to that given in Fact 1 for the Poisson network. [3]

[3] We believe, a more general result, regarding convergence of one-dimensional characteristics of arbitrary (regular in some sense) networks, to the Poisson limit, when $\sigma \to \infty$, can be worked out following arguments similar to these used in the proof of the random-translations convergence in [21, Theorem 9.4.II].

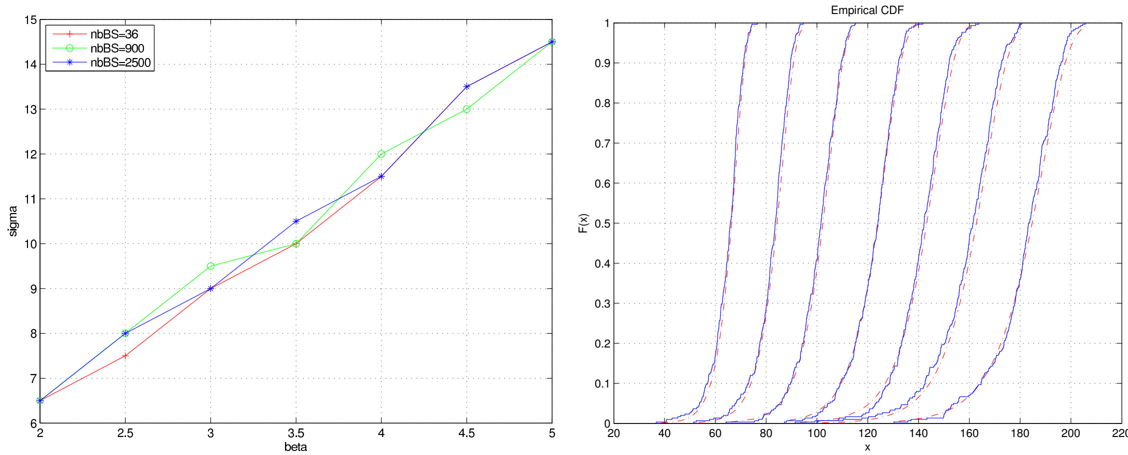

Fig. 1: Left: Critical values of $\sigma_{\mathrm{dB}(S)}$, above which, the empirical distribution of L^* in the hexagonal network cannot be distinguished from this for the "equivalent" Poisson model (cf. Section II-E). Right: Visual comparison of the cdf's for the hexagon model of $6 \times 6 = 36$ BS and the Poisson one for $\beta = 2, 2.5, 3, \ldots, 5$ (curves from left to right) for the corresponding critical values.

Consider the hexagonal network Φ_H^N of $N \times N$ BS located in the rectangle $[-N\Delta/2, N\Delta/2) \times [-N\sqrt{3}\Delta/4, N\sqrt{3}\Delta/4)$, where Δ is the distance between two adjacent stations, and serving users located in this rectangle. Note that the density of such a network is equal to $\lambda = 2/(\Delta^2\sqrt{3})$. In order to be able to neglect the boundary effects, let us assume the *toroidal metric* ("wrap around" the network, see [17] for details). Consequently, the distribution of $L^* = L^*(y)$ does not depend on the location y. A closed form expression for this distribution is not known, however, in the case of log-normal shadowing with sufficiently large values of $\sigma_{\mathrm{dB}(S)}$, this distribution can be approximated (fitted) by the corresponding expression for the infinite Poisson network given in Fact 1, and this even for quite small size N of the (finite) hexagonal network. To support this claim, we simulate the hexagonal network and compare the empirical cdf of L^* to that given in Fact 1 with the same parameters. We observe that the *supremum* (Kolmogorov) distance between the two cdf decreases in $\sigma_{\mathrm{dB}(S)}$. To make this observation more quantitative, we perform Kolmogorov-Smirnov (K-S) test (which is based on this distance; cf. [22]) and on Figure 1 (Left) we show the values of $\sigma_{\mathrm{dB}(S)}$, in function of β for $N = 6, 30, 50$, above which the K-S test does not allow to distinguish the empirical cdf for the hexagonal model (based on 300 observations) from the analytical (Poisson case) expression, at the 99% confidence level, for 9/10 realizations of the hexagonal network

Figure 1 (Right) visualizes the goodness of fit for these critical values of $\sigma_{\mathrm{dB}(S)}$ for $N = 30$ (i.e., 30×30 network).

III. Numerical Experiments

We will apply now the approach described in previous sections, using simulation and measurement regarding L^*, realized and collected in the cellular network of *Orange* in a certain large city in Europe,

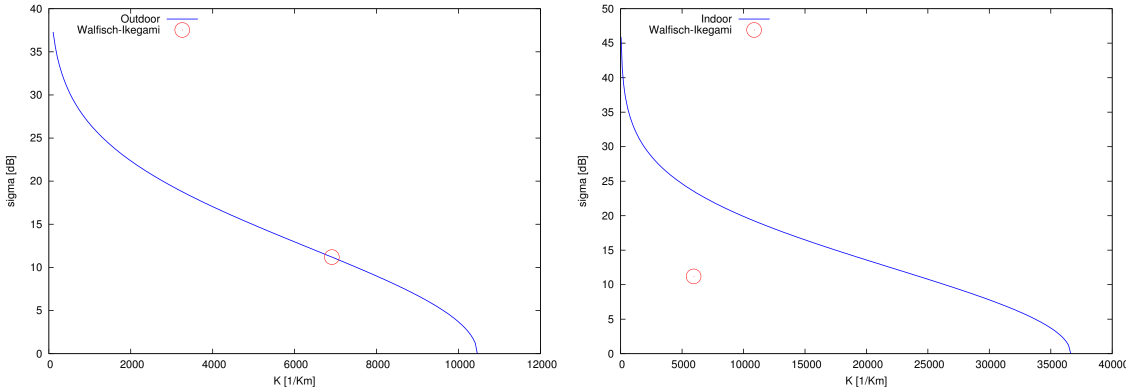

Fig. 2: Left: $\sigma_{\mathrm{dB}(S)}$ as function of the constant K estimated from *StarWave* (2.1GHz) data for outdoor, by the linear-regression method; estimated values are $\beta = 3.85$, $\tilde{K} = 10\,461\mathrm{km}^{-1}$. The red point designates the values obtained by conjunction of this approach with the COST Walfisch-Ikegami model. Right: similar estimation from the real data (80% indoor) collected in a 1.8GHz GSM network of *Orange*; $\beta = 3.64$. $\tilde{K} = 36\,622\mathrm{km}^{-1}$. The red point corresponds to the COST Walfisch-Ikegami model for outdoor scenario for this frequency.

with density $\lambda = 5.09\mathrm{km}^{-2}$. We shall deduce respectively the outdoor then the indoor propagation characteristics of this dense urban area.

A. Outdoor

The BS positions are these of the UMTS network of *Orange* in a certain large city in Europe, operating with the carrier frequency of 2.1GHz. The distribution of the outdoor loss L^* with the serving BS is obtained by simulations carried with *StarWave*, a propagation software developed by *Orange Labs*.[4] The linear fitting (1) of the empirical data obtained from these simulations gives the values $\beta = 3.85$ and $\tilde{K} = 10\,461\mathrm{km}^{-1}$; [5]. The curve given on Figure 2 (Left) shows all the couples of K and $\sigma_{\mathrm{dB}(S)}$ characterized by (2).

Following Remark 3, we consider also the COST Walfisch-Ikegami model for the same frequency 2.1GHz; [6]. This model gives the following values $\beta = 3.80$, $K = 6\,910\mathrm{km}^{-1}$ for the non-line-of-sight path-loss. Observe that the values of β obtained by the two approaches are close to each other, which may validate the novel approach for the data under consideration. Plugging the above value of K into (3) allows to derive the logarithmic standard deviation of the shadowing $\sigma_{\mathrm{dB}(S)} = 11.2\mathrm{dB}$.

[4]This tool uses detailed relief and building's databases and accounts for the diffraction, the guided propagation as well as the reflection on the relief.

[5]The 95%-confidence intervals are $\beta \in [3.34, 4.54]$, $\tilde{K} \in [3\,377, 32\,404]\mathrm{km}^{-1}$; the Kolmogorov distance between the empirical cdf and the estimated theoretical cdf is $D = 0.274$.

[6]The other network parameters are: BS antenna height 30m, mobile antenna height 1.5m, percentage of buildings 70%, nominal building height 25m, building separation 30m and street width 20m.

B. Indoor

We consider now the actual users' data collected in the GSM network of *Orange* operating with the carrier frequency 1.8GHz;[7]. The operator estimates that approximately 80% of users are in indoor conditions and the remaining 20% in outdoor. The linear fitting (1) gives $\beta = 3.64$, $\tilde{K} = 36\,622\text{km}^{-1}$;[8]. Figure 2 (Right) represents $\sigma_{\text{dB}(S)}$ in function of K for these estimates. We are not aware of any alternative model valid for indoor scenario, similar to the COST Walfisch-Ikegami model, to validate these estimates. The following approach to indoor scenario is often adopted.

Let us assume that indoor conditions do not modify the path-loss exponent β (which hence remains the same as for the outdoor communications in the same network) but modify only the values of the constants K and $\sigma_{\text{dB}(S)}$. More precisely, let us assume that the "overal" propagation loss can be decomposed as $(K_{out}r)^\beta/(S_{out} \times S_{in})$, where the constant K_{out} and the normalized log-normal S_{out} correspond to the outdoor scenario, and S_{in} is a log-normal random variable, independent of S_{out}, with some mean $\mathbf{E}[S_{in}]$ (not necessarily equal to one, reflecting additional mean indoor penetration loss) and logarithmic standard deviation $\sigma_{\text{dB}(S_{in})}$ (reflecting additional indoor loss variability). This model boils down to our previous model of Section II-A with $K = K_{out} \times K_{in}$, where $K_{in} = 1/\mathbf{E}[S_{in}]^{1/\beta}$ and $\sigma_{\text{dB}(S)} = \sigma_{\text{dB}(S_{out})} + \sigma_{\text{dB}(S_{in})}$.

Following this approach, we can use the COST Walfisch-Ikegami propagation model to obtain (for the carrier frequency 1.8GHz) $K_{out} = 5\,940\text{km}^{-1}$. It is also commonly believed that $\sigma_{\text{dB}(S_{out})}$ does not depend on the frequency. Hence we take our previous estimate $\sigma_{\text{dB}(S_{out})} = 11.2\text{dB}$. These values are depicted on Figure 2 (Right). The method does not allow to fix K_{in} and $\sigma_{\text{dB}(S_{in})}$ but gives only some bounds. Indeed, they should make K and $\sigma_{\text{dB}(S)}$ lie on the curve given on Figure 2 (Right), somewhere between the following two extremes: $K_{in} = 1$ with $\sigma_{\text{dB}(S_{in})} = 23.4\text{dB}$ (in which case indoor increases only the variance of the propagation loss) and $K_{in} = 4.09$ with $\sigma_{\text{dB}(S_{in})} = 0\text{dB}$ (in which case indoor increases only the mean propagation loss).

IV. Conclusion

We have proposed a new method allowing to estimate the parameters of the usual statistical model of the propagation loss, including shadowing, using data reported by the mobiles, regarding their propagation loss with respect to the serving BS. This complements existing statistical methods Okumura-Hata's or COST Walfisch-Ikegami model, by allowing for shadowing estimation.

[7] In fact the measurements concern the network operating on two frequency bands 1800MHz and 900MHz. Users connect first on 1800MHz, and in case of a problem switch to 900MHz. Therefore the reported data may lead to an underestimation of large values of the propagation loss.

[8] The 95%-confidence intervals are $\beta \in [3.42, 3.88]$, $\tilde{K} \in [21\,121, 63\,477]\text{km}^{-1}$; the Kolmogorov distance between the empirical cdf and the estimated theoretical cdf is $D = 0.119$. Note a better fit than in the case of the outdoor data, which is probably due to a larger value of $\sigma_{\text{dB}(S)}$.

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