QoS and network performance estimation in heterogeneous cellular networks validated by real-field measurements

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Outline

Introduction

- Cell load and the mean user throughput per cell are key QoS/network parameters in cellular data networks. They depend on (among other) :
  - traffic demand
  - base stations (BS) positioning
    - irregularity$\Rightarrow$ throughput vary across cells
  - inter-cell interference$\Rightarrow$ throughput in different cells interdependent
- We are also interested in the mean number of users per cell
- In this work we propose :
  - a semi-analytic approach to evaluate the spatial distributions of cell load, mean number of users and mean user throughput per cell in large heterogeneous cellular networks
  - validated by real-network measurements performed in operational networks

Network model

- BS ($X_1, X_2, \ldots \in \mathbb{R}^2$) locations modeled by a homogeneous Poisson point process $\Phi = \sum_{k\in\mathbb{N}} \delta_{X_k}$ on $\mathbb{R}^2$ with intensity parameter $\lambda \in \mathbb{R}_+^*$
- Each BS X_n is characterized by a transmitting power $P_n \in \mathbb{R}_+^*$
- BS transmitting power $P_1, P_2, \ldots$ are *i.i.d.* (independent and identically distributed) *marks* of the point process Φ
- Propagation loss $l(x)$ due to distance x is given by

$$\ell(x) = (K\,|x|)^{\beta}, \quad x \in \mathbb{R}^2 \tag{1}$$

  where $K > 0$ and $\beta > 2$ are given constants.
- *Shadowing* between a given base station X_n and all locations $y \in \mathbb{R}^2$ is modeled by some random field $\{\mathbf{S}_n(y - X_n)\}_{y\in\mathbb{R}^2}$
- The shadowing random fields $\{\mathbf{S}_n(\cdot)\}_{n\in\mathbb{N}^*}$ and the transmitting powers $\{P_n\}_{n\in\mathbb{N}^*}$ are assumed independent.

Network model

Consequently,

- The received power at location $y \in \mathbb{R}^2$ from BS X_n equals

$$L_n^{-1}(y) = \frac{P_n \mathbf{S}_n (y - X_n)}{\ell (y - X_n)}, \quad n \in \mathbb{N}^* \tag{2}$$

- We assume that each base station serves the zone (called *cell*) where its signal is the strongest one :

$$V(X_n) = \left\{ y \in \mathbb{R}^2 : L_n(y) \leq \min_{k \in \mathbb{N}^* \setminus \{n\}} L_k(y) \right\} \tag{3}$$

Service model

- Locations of BS don't evolve in time
- In HSDPA and LTE networks a given base station transmits only if it has at least one user to serve.
- We take into account whether an interfering BS X_k is idling or not, multiplying its powers P_k by the *probability* $p(X_k)$ that it is not idle in the steady state of users' arrivals and departures.
- Thus the SINR equals

$$\mathrm{SINR}\left(y,\Phi\right) = \frac{L_n^{-1}\left(y\right)}{N + \sum_{k\in\mathbb{N}^*\setminus\{n\}} p(X_k)L_k^{-1}\left(y\right)} \tag{4}$$

for every $y \in V\left(X_n\right)$, where N is the noise power.

Service model

- Bit-rate of a user located at y when served alone by its BS, called *peak bit-rate*, $R\left(\mathrm{SINR}\left(y,\Phi\right)\right)$
- The *peak bit-rate* at location y has the following form [7, Equation (3.169)]

$$R\left(\mathrm{SINR}\right)=bW\mathbf{E}\left[\log_2\left[\det\left(I+\mathrm{SINR}HH^*\right)\right]\right]$$

where I is identity matrix, W is the frequency bandwidth, b is a correction factor accounting for a practical performance of wireless channel, H is a random matrix representing fading, and $\mathbf{E}[\cdot]$ is the mathematical expectation with respect to fading. In the particular case of SISO (Single Input Single Output) channel, we have

$$R\left(\mathrm{SINR}\right)=bW\mathbf{E}\left[\log_2\left(1+\mathrm{SINR}\left|H\right|^2\right)\right] \tag{5}$$

where H is the fading random variable.

Traffic model and service policy

- We shall consider variable bit-rate (VBR) traffic : users require to transmit some volume of data at a bit-rate decided by the network
- There are γ arrivals per surface unit and per time unit
- Each user arrives at a location uniformly distributed and requires to download a random volume of data of mean $1/\mu$ bits
- Arrival locations, inter-arrival durations as well as the data volumes are assumed independent
- Users don't move during their calls
- *Traffic demand* per surface unit

$$\rho = \frac{\gamma}{\mu} \text{ bit/s/km}^2$$

Service policy

- The traffic demand in cell $X_n \in \Phi$ equals

$$\rho(X_n) = \rho\,|V(X_n)|, \quad n \in \mathbb{N}^* \ \text{bit/s} \tag{6}$$

  where $|V(X_n)|$ is the surface of area of the cell $V(X_n)$.
- We shall assume that each user in a cell gets an equal portion of time for his service. Thus, a base station located at X serves k users located at $y_1, y_2, \ldots, y_k \in V(X)$ then the bit-rate of the user located at y_j equals

$$\frac{1}{k} R\left(\mathrm{SINR}\left(y_j, \Phi\right)\right), j \in \{1, 2, \ldots, k\}$$

  .

Local characteristics model [4]

- Service in cell $V(X)$ is *stable* when

$$\rho(X_n) < \rho_{\mathrm{c}}(X_n) := \frac{|V(X_n)|}{\int_{V(X_n)} 1/R(\mathrm{SINR}(y,\Phi))\,dy} \tag{7}$$

called *critical traffic* : harmonic mean of the peak bit-rates

- Mean *user throughput*

$$r(X_n) = \max(\rho_{\mathrm{c}}(X_n) - \rho(X_n), 0) \tag{8}$$

- Mean *number of users*

$$N(X_n) = \frac{\rho(X_n)}{r(X_n)} \tag{9}$$

- Probability that BS is not idling

$$p(X_n) = \min(\theta(X_n), 1) \tag{10}$$

where $\theta(X_n) = \dfrac{\rho(X_n)}{\rho_{\mathrm{c}}(X_n)} = \rho \displaystyle\int_{V(X_n)} 1/R(\mathrm{SINR}(y,\Phi))\,dy$

$$\tag{11}$$

called *cell load*

Motivation of this work

Plot $r(X)$ as function of $\rho(X)$ (measurements for different cells during different hours of the day) : no apparent relation between local characteristics

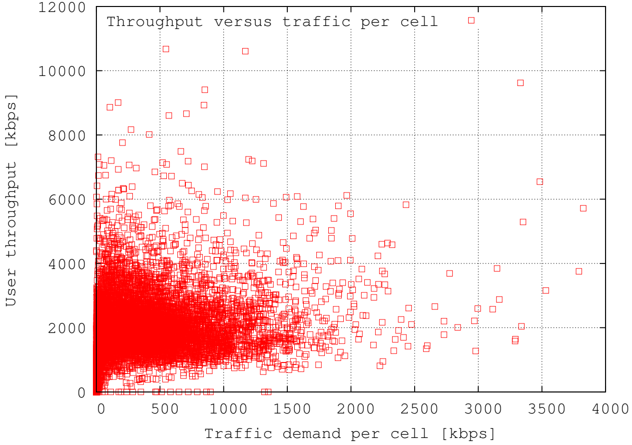

Figure 1:

Dependence between cells (cell-load equations)

Using Equations (11) and (7) respectively, we deduce that the cell load of BS X_n equals

$$\begin{aligned}
\theta(X_n) &= \frac{\rho(X_n)}{\rho_{\mathrm{c}}(X_n)} = \rho \int_{V(X_n)} R^{-1}(\mathrm{SINR}(y, \Phi))\, dy \\
&= \rho \int_{V(X_n)} R^{-1}\left(\frac{L_n^{-1}(y)}{N + \sum_{k \neq n} p(X_k) L_k^{-1}(y)} \right) dy \\
&= \rho \int_{V(X_n)} R^{-1}\left(\frac{L_n^{-1}(y)}{N + \sum_{k \neq n} \min(\theta(X_k), 1) L_k^{-1}(y)} \right) dy \qquad (12)
\end{aligned}$$

All parameters expressed via cell load

- Note here again that the the load of each BS depends on the loads of all other BS in the network.
- From the result presented in [6] we can conclude the uniqueness of the solution of (12) with $\theta(X_n) < 1$, ($\forall n \in \mathbb{N}^*$) i.e. if such a solution exist then it is unique.
- Note also that the loads which are equal or greater than one indicate unstable cells.

On the other hand, we can express the mean number of users (9) and the mean user throughput (8) in each cell as function of its load and traffic demand

$$N(X_n) = \left[\max\left(\frac{1}{\theta(X_n)} - 1, 0\right)\right]^{-1}, \quad r(X_n) = \frac{\rho(X_n)}{N(X_n)} \quad (13)$$

Discrete users' positions

- We generate users' positions over the network as a homogeneous Poisson point process of density 30λ and approximate the integral in (12) by the corresponding discrete sum.
- The shadowing random variables for the different user positions are generated as i.i.d. log-normal random variables with *logarithmic-standard deviation* σ_S (expressed in dB).
- Note that the auxiliary users' point process permits not only to evaluate numerically the integral in (12), but also permits to account for spatial correlation of shadowing ; the mean spatial decorrelation distance being the average distance between two user locations.

Constant power model

In order to evaluate the effect of the variability of the BS powers, we shall compare the results of the above model with a model where all the BS emit the same power. In this latter, all the BS emit the power

$$\tilde{P}_n = E\left[P_1\right], \quad n \in \mathbb{N}^*$$

and the shadowing equals

$$\tilde{\mathbf{S}}_n(y - X_n) = \frac{P_n \mathbf{S}_n\left(y - X_n\right)}{E\left[P_n\right]}, \quad n \in \mathbb{N}^* \tag{14}$$

so that the received power (2) remains as the original model.

Numerical results

- We compare the CDF (cumulative distribution function) of cell load, mean number of users and mean user throughput of the different base stations at a given hour in an operational HSDPA network (within two different cities) to the results of the proposed method.
- The raw data are collected using a specialized tool which is used by operational engineers for network maintenance.
- This tool measures several parameters for every base station 24 hours a day on TTI time-scale.
- We have also the BS coordinates which permit to estimate the intensity λ of BS per unit surface.
- We choose one hour during the day and estimate the corresponding empirical CDF of the QoS parameters.

Numerical setup for simulation

- We generate a Poisson process of BS with intensity λ over a disc of radius
$$D = 10\sqrt{\frac{1}{\pi\lambda}}$$
- The distance coefficient in (1) equals $K = 7117\text{km}^{-1}$, the path loss exponent $\beta = 3.8$.
- The shadowing standard deviation equals $\sigma_S = 8\text{dB}$.
- The frequency bandwidth equals $W = 5\text{MHz}$ and the noise power is $N = -96\text{dBm}$. We take $b = 0.3$ in the peak bit-rate expression (5) for HSDPA.
- We consider three-sectorial antennas with azimuths π, $\pi + 2\pi/3$ and $\pi - 2\pi/3$ and antenna pattern described in [1, Table A.2.1.1-2].

Numerical setup for simulation - powers

- We assume that the transmitting power P_n has a log-normal distribution of logarithmic-standard deviation σ_P.
- In order to justify this model, we give the empirical CDF of transmitting powers in dB estimated from measurements in the operational network on Figure 2. This figure shows that this CDF may be approximated by a normal distribution with standard deviation $\sigma_P = 5.3$dB.
- The mean transmitting power of each BS including a global antenna gain equals $E[P_n] = 60$dBm. A 10% fraction of this power is used by the pilot channel.
- In the constant power model described in Section 6, each BS emits a constant power $\tilde{P}_n = 60$dBm and the shadowing (14) has a log-normal distribution of standard deviation

$$\sigma_{\tilde{S}} = \sqrt{\sigma_S^2 + \sigma_P^2} \simeq 9.6\text{dB}$$

Numerical setup for simulation - powers

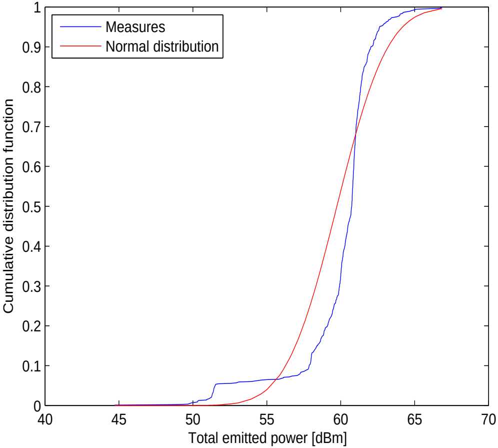

Figure 2: CDF of BS powers in the operational network in the downtown of a big city (blue) and normal distribution approximation (red).

Poisson network hypothesis

- Regarding spatial pattern of BS, we use Poisson model.
- The idea of using Poisson process to model cellular network already exist in the literature, see for example [5].
- Moreover, it is shown in [2] that starting from any deterministic pattern of BS (including the regular Hexagonal one), when the shadowing variance becomes sufficiently high, the radio parameters (such as the propagation loss with the serving base station) converge to those of a Poisson model.
- This is a reasonable assumption, especially for urban environment and/or indoor position of receiver.
- Second, deployed networks don't follow a regular spatial pattern, especially not those in urban and suburban environments, so the mentioned convergence is faster.

Poisson network hypothesis

- For example, in some urban areas in Europe, the geographical pattern of base stations is nearly Poissonian which is checked in Figure 3 using Ripley's L-function $L(r) = \sqrt{K(r)/\pi}$, where $K(r)$ is Ripley's K-function [3].
- The other sources of irregularities, as for example non-uniform traffic demand, are not considered in the present work. Consequently, we will consider networks or parts of a network where we can assume uniform spatial traffic demand (e.g. downtown of a big city or a typical rural area).

Ripley's function

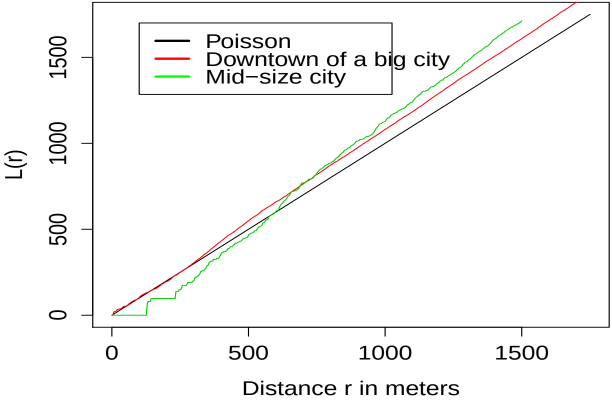

Figure 3: Ripley's L-function of two patterns of base stations which are nearly Poissonian. Note that mid-size city exhibits less irregularity than the downtown of a big city.

Results

- Figures 4, 5 and 6 show the spatial distribution (across different cells) of the cell load, mean number of users per cell and the mean user throughput in the network deployed in the downtown of a big city.
- Recall that these metrics represent, themselves, the steady-state (averaged over time) performance characteristics of individual cells.

Cell load : downtown of a big city

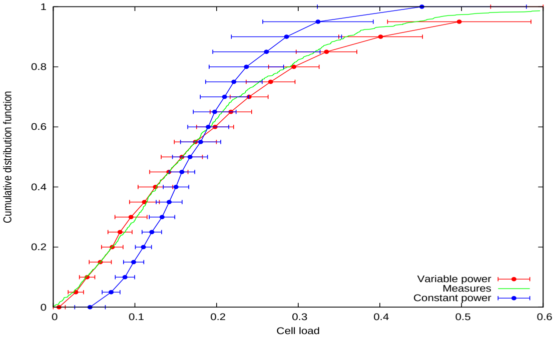

Figure 4: CDF of cell load for the downtown of a big city obtained either from the variable power model, from real-field measurements, or from the model where the emitted powers are assumed constant.

Mean number of users : downtown of a big city

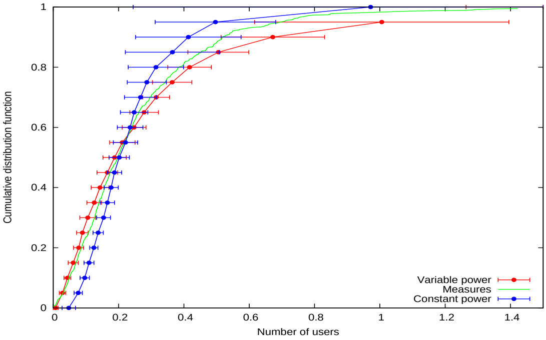

Figure 5: CDF of the mean number of users for the downtown of a big city.

Mean user throughput : downtown of a big city

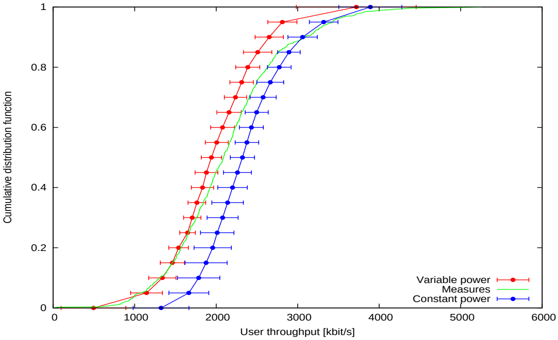

Figure 6: CDF of the mean user throughput for the downtown of a big city.

Model input values

- Analogous characteristics regarding the network in a mid-size city are presented on Figures 7, 8 and 9.
- The estimated network density and the traffic demand in the downtown of a big city are, respectively, $\lambda = 4.62\text{km}^{-2}$ and $\rho = 483\text{kbit/s/cell}$.
- Analogous values for the mid-size city are $\lambda = 1.27\text{km}^{-2}$ and $\rho = 284\text{kbit/s/cell}$. Note that in the latter scenario the traffic demand is smaller, but the network is less dense and more regular (cf Figure 3).
- We use these values as input parameters for our model.

Cell load : mid-size city

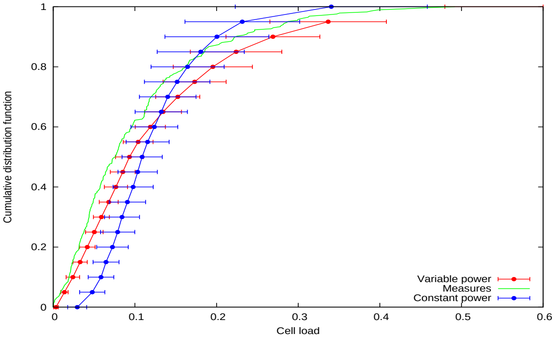

Figure 7: CDF of cell load for mid-size city obtained either from the variable power model, from real-field measurements, or from the model where the emitted powers are assumed constant.

Mean number of users : mid-size city

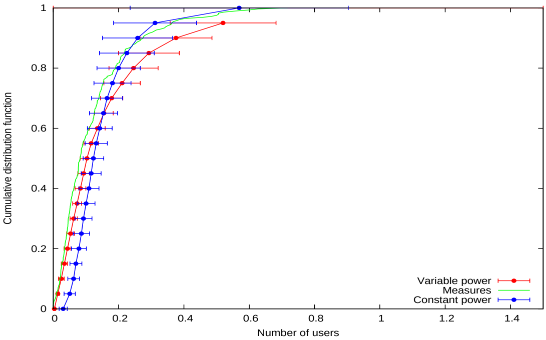

Figure 8: CDF of the mean number of users for the mid-size city.

Mean user throughput : mid-size city

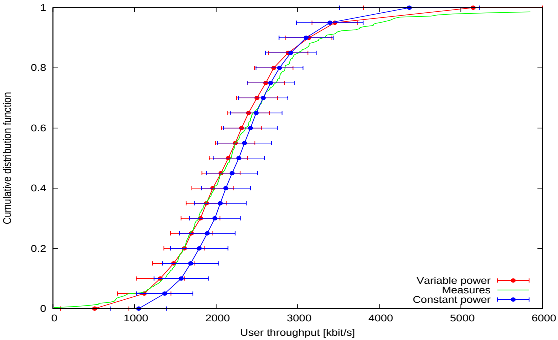

Figure 9: CDF of the mean user throughput for the mid-size city.

Results explanation

- All figures present the distributions estimations from our model as well as the real-field measurements. For seek of comparison, we present also on the figures the results obtained in the model where the emitted powers are assumed constant.
- The simulation curves represent the means over ten repeated network simulations, with the horizontal bars giving the standard deviation of this averaging. In what follows we discuss the presented results in more details.

Results analysis

- In general we see a good agreement between real field measures and the model analysis with randomized emitted power.
- Under the constant power assumption the model predicts well the median of the cell load and the mean number of users but fails to match the spatial distribution of these characteristics.
- Clearly, the spatial variability of power creates more spatial heterogeneity of these characteristics in the network.
- Regarding the mean user throughput the constant power assumption fails to predict even the median.
- Extensions of the model, e.g. letting it account for further sources of disparity in the deployed networks (e.g. different heights of antennas) could perhaps improve the quality of prediction.

Conclusion

- The approach based on stochastic geometry in conjunction with queueing and information theory is developed
  - In order to evaluate user's QoS metrics and network parameters
  - Spatial distributions of cell load, mean number of users and mean user throughput are derived
- We validate the proposed approach by showing that it allows to predict the performance of a real network
- Open questions
  - Theoretical : stability of spatially and, more difficult, space-time dependent processor sharing queues
  - Investigate the effects of other sources of disparity in a network

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