Study of a Key Factor for Performance Evaluation of Wireless Cellular Networks: The f-Factor

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***Abstract*—The f-factor, which is roughly the interference to signal power ratio, plays a crucial role in the performance evaluation of wireless cellular networks. The objective of the present paper is to study the properties of the f-factor and establish approximations for it which we compare to previously proposed approximations.**

We consider the *hexagonal network model*, where the base stations are placed on a regular hexagonal grid which may be infinite. The propagation loss is assumed to be a power of the distance between the transmitter and the receiver. In this context, we build a *reference* method to calculate the f-factor to which previously proposed approximations as well as a new one are compared. It is shown that the previous approximations are not always close to the reference. One should choose the approximation carefully since the performance of cellular networks depend strongly on the f-factor. The results in our paper help to make the appropriate choice. This is particularly important for operational needs as for example dimensioning a real network.

***Index Terms*—Radio communication, Broadband communication, Communication system performance, Interference, Geometry.**

I. Introduction

When we attempt to evaluate the performance of wireless cellular networks, the so-called *f-factor* or *interference factor*, appears naturally. Indeed, it may be defined as the ratio of the interference to the signal power received at a given location when all the base stations transmit the same power. But its importance is not limited to this restrictive case. It plays an important role in the resource (power and bandwidth) allocation problem for broadband systems which use either Code-Division Multiple Access (CDMA) or Orthogonal Frequency-Division Multiple Access (OFDMA) as shown in [6] and [4] respectively. It is shown there that the *resource allocation* problem admits a solution when a simple condition on the bit-rates of the users expressed in terms of the f-factor is satisfied. Moreover, it is shown that the *QoS* perceived by the users (blocking probability, delay, throughput) may be evaluated analytically by using appropriate queueing models when we know the f-factor. This leads to efficient and rapid capacity planning and *dimensioning* methods.

Thus the f-factor plays a crucial role in the performance evaluation of wireless cellular networks. The *objective* of the present work is to study the properties of the f-factor and to establish approximations for it which we compare to previously proposed approximations.

Note first that the f-factor depends on the geometry of the network (base stations positions). More precisely, the effect of the network geometry on the performance may be characterized through its effect on the f-factor. For this reason, it is sometimes called the *geometric* factor. On the other hand, the number of base stations in real cellular networks becomes increasingly large due to the continual extension of their coverage and also due to the densification induced by the traffic increase. Thus it is important to study *large cellular networks*, and eventually the limit case when the network is infinite.

The remaining part of this paper is organized as follows. In the following two subsections of the introduction we present briefly the related work and describe our model. The basic properties of the f-factor (and in particular a *reference* method to calculate it) are given in Section II. In Section III we study the variations of the f-factor versus user location. Previously proposed approximations as well as a new one are compared to the reference value in Section IV.

A. Related works

The importance of the f-factor as a key factor in cellular networks has been recognized since long time in many publications (see for example [9], [15]). The paper [14] focuses on the f-factor average over the cell, and in particular the effect of shadowing on this average. Frequently the f-factor is computed by simulations (see for example [12]). In [11] the probability distribution function of the f-factor is studied.

More recently, many papers [1], [5], [6], [8] propose explicit approximations of the f-factor and its moments (mean and variance). Nevertheless, the properties of the f-factor are not widely known, and no comparison between its approximations have been published yet.

B. Model description

Consider a network composed of base stations (BS) located on the plane $\mathbb{R}^2$. We denote by $L_{u,m}$ the propagation-loss between BS u and location m. We shall only account for the *distance effect* (no shadowing), that is $L_{u,m}$ is some function of the distance between u and m. Typically we shall consider

$$L_{u,m} = (K\,|u-m|)^{\eta} \tag{1}$$

where $K > 0$ and $\eta > 2$ are two constants; and $|u-m|$ designates the distance between u and m.

Each BS u serves some geographic zone called *cell*. We denote $m \in u$ to say that the location m is served by the BS u. Hence we use the same letter to designate the BS and its cell. We assume first that each BS is equipped with an omnidirectional antenna. Then we extend the results to sectorial antennas.

We consider the most popular *hexagonal network model*, where the base stations are placed on a regular hexagonal grid which may be infinite. The cell of BS u is defined by $\{m \in \mathbb{R}^2; |u-m| \leq |v-m| \text{ for all BS } v\}$.

1) Problem formulation: Fix some base station u. The corresponding *f-factor* is a function of the location m defined by

$$f(m) = \sum_{v \neq u} \frac{L_{u,m}}{L_{v,m}}, \quad m \in u$$

where the summation is over all BS v different from u.

We aim to study the properties of the function $f(m)$ and to establish a suitable approximation for it.

II. Basic properties

We begin by studying f as function of the propagation-loss parameters K and η (see Equation (1)). First observe that the f-factor is independent of the constant K, thus we may take $K=1$ without loss of generality.

Let Δ be the distance between two adjacent base stations. If we make a homothecy, then all the distances are multiplied by the same factor, thus the f-factor doesn't change. Therefore we can fix $\Delta = 1$ without loss of generality.

For a fixed location $m \in u$, the f-factor is a non-increasing function of η. (Indeed, fix $m \in u$. For each $v \neq u$, $|u-m| \leq |v-m|$ thus $L_{u,m}/L_{v,m} = (|u-m|/|v-m|)^{\eta}$ is a non-increasing function of η; from which the desired result follows.)

Since all the base stations play a symmetric role, we consider a given one, say u, and take its position as the origin of the coordinate system. The corresponding cell is a Hexagon of center u.

A. Decomposition

We shall first observe that, for a given BS u, the f-factor may be *decomposed* into a sum of terms; each one corresponding to the contribution of the BSs located on a hexagon admitting u as center. The first hexagon corresponds to the six closest BSs to u.

More precisely, note that when the BS u is fixed, the other BSs are located on successive hexagons having u as center and having increasing radii (See Fig. 1). These hexagons are called *levels* and denoted $\mathcal{L}_1, \mathcal{L}_2, \ldots$. We may decompose $f(m)$ over the different levels as follows (similarly to [11, Eq. (41)])

$$f(m) = \sum_{k \geq 1} f_k(m)$$

where

$$f_k(m) := \sum_{v \in \mathcal{L}_k} \frac{L_{u,m}}{L_{v,m}}.$$

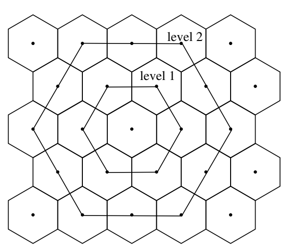

Fig. 1. The first two levels $\mathcal{L}_1$ and $\mathcal{L}_2$.

In the following Lemma we shall express the *contribution* of the BSs located on a given level in terms of the contribution of the BSs located on the first one.

Lemma 1: We take a BS u as the origin of the coordinate system and identify $\mathbb{R}^2$ with the complex plane $\mathbb{C}$. We have, for all $m \in u$, $f_1(m) = |m|^{\eta} \sum_{l=0}^{5} \left| m - e^{i\frac{l\pi}{3}} \right|^{-\eta}$. Moreover,

$$f_2(m) = f_1\left(\frac{m}{2}\right) + f_1\left(\frac{m}{\sqrt{3}} e^{-i\frac{\pi}{6}}\right)$$

and more generally,

$$f_k(m) = \sum_{l=0}^{k-1} f_1\left(\frac{m}{\sqrt{k^2+l^2-kl}} e^{-i\frac{l\pi}{3k}}\right), \quad k \geq 2.$$

Proof: The expression of $f_1(m)$ is immediate. An inspection of Fig. 1 shows that $\mathcal{L}_k$ comprises $6k$ BSs. We may decompose $\mathcal{L}_k$ into k groups of BSs indexed by $l = 0, \ldots, k-1$; each group l is composed of 6 BSs being at distance $\sqrt{k^2+l^2-kl}$ from the center. Consider the polar coordinates with respect to the central BS as origin. Then the angular coordinate of the first BS of each group l is $\frac{l\pi}{3k}$. Recalling that f is invariant by homothecy, we deduce that the contribution of the l-th group is $f_1\left(\frac{m}{\sqrt{k^2+l^2-kl}} e^{-i\frac{l\pi}{3k}}\right)$ which finishes the proof. ∎

B. Calculus precision

Obviously, we have $f(m) \geq \sum_{k=1}^{n} f_k(m)$ for all integer n. We may estimate the f-factor with the sum in the right-hand side of the previous inequality. But how many terms are necessary to guarantee some precision? The following approximation will help to answer this question.

Approximation 1: We have

$$f_k(m) \simeq k^{-(\eta-1)} f_1(m)$$

and

$$f(m) - \sum_{k=1}^{n} f_k(m) \simeq \left[\zeta(\eta-1) - \sum_{k=1}^{n} k^{-(\eta-1)}\right] f_1(m)$$

where ζ is the *Riemann zeta function* given by $\zeta(x) = \sum_{k=1}^{\infty} k^{-x}$.

Proof: An inspection of Fig. 1 shows that $\mathcal{L}_k$ comprises $6k$ BSs; six of which are at distance k from the center and the other $6(k-1)$ BSs are at distances slightly less than k. Let $\mathcal{H}_k$ be the set of the 6 BSs situated at distance k from the center. Then making the approximation as in [9] that, all the base stations of level k are at the distance k, we get

$$\begin{aligned} f_k(m) &\simeq k \sum_{v \in \mathcal{H}_k} \frac{L_{u,m}}{L_{v,m}} \\ &\simeq k\left(k^{-\eta} \sum_{v \in \mathcal{L}_1} \frac{L_{u,m}}{L_{v,m}}\right) = k^{-(\eta-1)} f_1(m). \end{aligned}$$

Thus we get the approximation

$$\begin{aligned} f(m) - \sum_{k=1}^{n} f_k(m) &= \sum_{k \geq n+1} f_k(m) \\ &\simeq \sum_{k \geq n+1} k^{-(\eta-1)} f_1(m) \\ &= \left[\zeta(\eta-1) - \sum_{k=1}^{n} k^{-(\eta-1)}\right] f_1(m). \end{aligned}$$

∎

Using the Approximation 1, we deduce that the relative error if we estimate $f(m)$ with $\sum_{k=1}^{n} f_k(m)$ is

$$\begin{aligned} \frac{f(m) - \sum_{k=1}^{n} f_k(m)}{f(m)} &\simeq \frac{\zeta(\eta-1) - \sum_{k=1}^{n} k^{-(\eta-1)}}{\zeta(\eta-1)} \\ &= 1 - \frac{\sum_{k=1}^{n} k^{-(\eta-1)}}{\zeta(\eta-1)}. \end{aligned}$$

We shall consider that the f-factor is calculated sufficiently precisely when the relative error is less than some given ϵ sufficiently small. Therefore, for each η, we consider a number of levels equal to the smallest value of n such that the right-hand side of the above equation is less than ϵ.

Example 1: We are particularly interested in $\eta \in [3,5]$ which comprises the most frequent values in practical cellular networks. Fig. 2 shows the number of levels n as function of the propagation exponent $\eta \in [3,5]$ for different values of the precision $\epsilon = 0.01, 0.05, 0.1$. We observe that n is decreasing with η. In other words, the calculus effort to calculate the f-factor with a given precision is decreasing with the propagation exponent. Moreover n may be large when both η and ϵ are small.

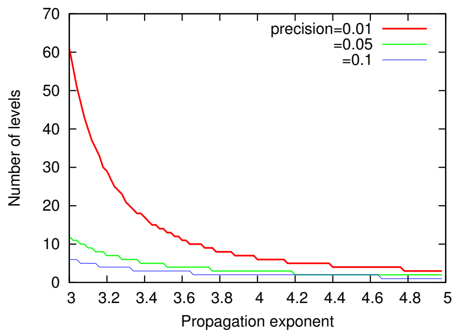

Fig. 2. Number n of levels as function of the propagation exponent η for different precisions $\epsilon = 0.01, 0.05, 0.1$.

From now on we consider as *reference* value the f-factor calculated with the number of levels corresponding to the precision $\epsilon = 0.05$.

III. Variations versus user location

A. f-factor versus user location

We take a BS u as the origin of the coordinate system. Each location $m \in u$ may be viewed as a complex number $re^{i\theta}$ where (r, θ) are the polar coordinates of m. Then $f(m)$ may be viewed as function the polar coordinates r and θ.

For each fixed r, the f-factor is a periodic function of θ with period $\pi/3$ (which is due to the same periodicity of the positions of the BSs $v \neq u$) and symmetric around $\ell\pi/6$ (for all $\ell \in \{0, 1, \ldots, 11\}$). If f is differentiable with respect to θ, then it admits an extremum at $\ell\pi/6$. We will see numerically that it is in fact a maximum at $\ell\pi/3$ and a minimum at $(2\ell + 1)\pi/6$.

Example 2: We take a propagation exponent $\eta = 3.38$ (which is a typical value in urban areas). Fig. 3 represents $f(m)$ as function of the user location m with cartesian coordinates (x, y) related to polar coordinates by $x + iy = re^{i\theta}$. We observe that the f-factor variation with respect to the angle θ is less important than that with respect to the distance r between the user and its serving BS. We calculate numerically the f-factor at cell edge and observe that it is maximum for $\theta = 0$ and minimum for $\theta = \pi/6$ (equal to 2.2 and 1.8 respectively).

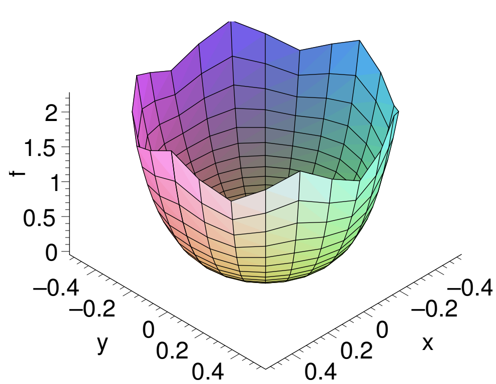

Fig. 3. f-factor as function of user location m of cartesian coordinates (x, y).

B. Average over the angle

Recall the observation made in Example 2 that the f-factor variation with respect to the angle θ is less important than that with respect to the distance between the user and its serving BS. Thus in some applications, it may be interesting to assume that the f-factor depends only on this distance. In this case, we take as representative value of the f-factor (at a given distance r) its average over the angle θ, that is

$$\bar{f}(r) = \frac{1}{2\pi} \int_0^{2\pi} f\left(re^{i\theta}\right) d\theta = \frac{6}{\pi} \int_0^{\pi/6} f\left(re^{i\theta}\right) d\theta \qquad (2)$$

where the second equality is due to the periodicity of f (and its symmetry around $\theta = \frac{\pi}{6}$). We have already observed that for fixed distance r, the f-factor is maximum for $\theta = 0$ and minimum for $\theta = \frac{\pi}{6}$. This suggests the approximation

$$\bar{f}(r) \simeq f\left(re^{i\pi/12}\right) \qquad (3)$$

Numerical calculus presented in the following example show that the above approximation is good.

Example 3: Fig. 4 plots the functions $\bar{f}(r)$ and $f\left(re^{i\pi/12}\right)$ of the distance r for $\eta = 3, 4, 5$. We observe that the approximation (3) is good.

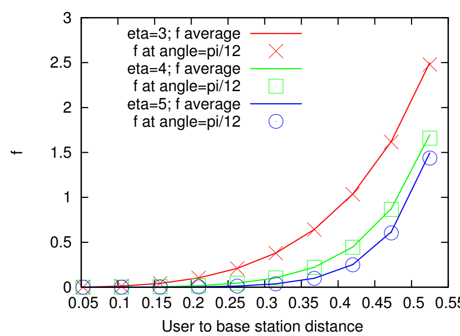

Fig. 4. The functions $\bar{f}(r)$ and $f\left(re^{i\pi/12}\right)$ versus the distance r for $\eta = 3, 4, 5$.

C. Average over the cell

In some applications we are interested in the average of the f-factor over the cell, that is

$$\bar{f} = \frac{1}{S_u} \int_u f(m)\, dm$$

where S_u designates the surface of the cell u.

In order to simplify the calculus, we approximate the hexagonal cell with the (virtual) disc whose area is equal to that of the hexagon. This is illustrated in Fig. 5. We define the *cell radius* R as the radius of this disc.

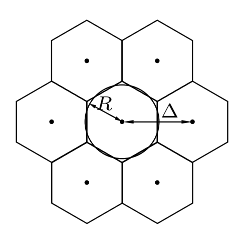

Fig. 5. Hexagon to disc approximation.

Lemma 2: [6, Lemma 1 p.10] The cell radius R is related to the distance Δ between two adjacent hexagons by

$$R = \Delta\sqrt{\frac{\sqrt{3}}{2\pi}} \qquad (4)$$

which gives numerically $R \simeq 0.525\,\Delta$.

Thus we get the following approximations

$$\bar{f} \simeq \frac{1}{\pi R^2} \int_0^R \int_0^{2\pi} f\left(re^{i\theta}\right) r dr d\theta$$
$$= \frac{2}{R^2} \int_0^R \bar{f}(r) r dr \simeq \frac{2}{R^2} \int_0^R f\left(re^{i\pi/12}\right) r dr$$

It is not difficult to calculate the integral in the right-hand side of the above equation. Nevertheless, we aim to get insight on how $\bar{f}$ varies as function of η and to establish an explicit approximation of this function.

Example 4: It is shown in [2] for the so-called Poisson-Voronoi model (which is different from the hexagonal one considered in the present study) that $\bar{f} = 2/(\eta - 2)$. This suggest to look for an approximation of $\bar{f}$ in the hexagonal model of the form $\bar{f} \simeq a/(\eta - 2)$ for some constant a to be calculated. We make this calculus by making a least square correlation between the values of $\bar{f}$ for $\eta \in [3, 5]$ and the function $1/(\eta - 2)$. This gives

$$\bar{f} \simeq \frac{0.91}{\eta - 2} \tag{5}$$

For some applications, we also need to calculate $\overline{f^2} := \frac{1}{S_u} \int_u f(m)^2 dm$ and $\overline{lf} := \frac{1}{S_u} \int_u \frac{L_{u,m}}{L(R)} f(m) dm$ where $L(R) = (KR)^\eta$. Similarly to $\bar{f}$, we get the following approximations

$$\overline{f^2} \simeq \frac{0.69}{\eta - 2} + \frac{0.64}{(\eta - 2)^2}$$

and

$$\overline{lf} \simeq \frac{0.6}{\eta - 2}$$

Fig. 6 shows $\bar{f}$, $\overline{f^2}$ and $\overline{lf}$ as functions of η as well as their approximations given in the above three equations. This figure shows that the approximations are good enough.

Applying Equation (5) for $\eta = 3, 4, 5$ we get $\bar{f} = 0.91, 0.45, 0.30$ respectively, whereas from [14, Table II] we get $\bar{f} = 0.77, 0.44, 0.30$ respectively. Note that there is a good agreement between our result and that of [14, Table II] for $\eta = 4, 5$. The disagreement for $\eta = 3$ is due to the fact that [14, Table II] considers two levels of interfering BSs which is too small when $\eta = 3$ (see Example 1).

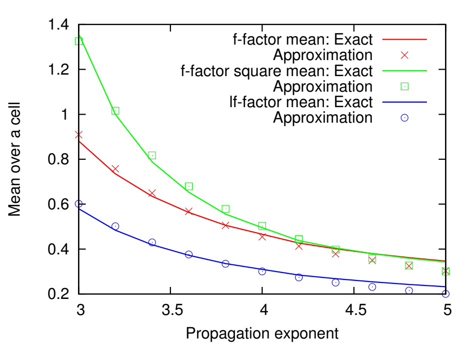

Fig. 6. $\bar{f}$, $\overline{f^2}$ and $\overline{lf}$ as functions of the propagation exponent η as well as their approximations.

IV. Approximations

In order to get an approximation of the f-factor, we consider the number of levels corresponding to a precision $\epsilon^{\mathrm{a}} = 0.1$ and denote it by $f^{\mathrm{a}}(m)$.

Example 5: We take a propagation exponent $\eta = 3.38$. Figure 7 (left) represents $f^{\mathrm{a}}(m)$ as function of the user location m. Note that $f^{\mathrm{a}}(m)$ captures the essential effects observed for $f(m)$ (see Example 2). Fig. 7 (right) represents the relative error $1 - f^{\mathrm{a}}(m)/f(m)$ as function of the user location m. Observe that the relative error is comprised between 0.03 at cell edge and 0.06 at cell center. (Observe also that at cell edge the relative error depends on θ, but it remains approximately equals to 0.03.)

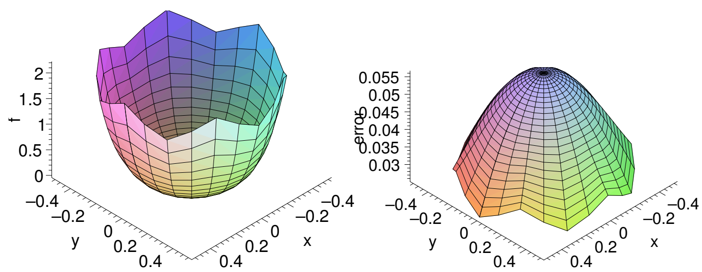

Fig. 7. Left: f-factor approximation $f^{\mathrm{a}}(m)$ as function of the user location $m = (x, y)$. Right: Relative error $1 - f^{\mathrm{a}}(m)/f^{\mathrm{p}}(m)$ as function of the user location $m = (x, y)$.

We may approximate $f\left(re^{i\pi/12}\right)$ in the right-hand side of (3) by $f^{\mathrm{a}}\left(re^{i\pi/12}\right)$, thus we get

$$\bar{f}(r) \simeq f^{\mathrm{a}}\left(re^{i\pi/12}\right) \tag{6}$$

In [6, p.212] the following approximation for the f-factor, denoted $f^{\mathrm{b}}(r)$, is proposed

$$f^{\mathrm{b}}(r) = \zeta(\eta - 1)\left(\frac{L(r)}{L(\Delta - r)} + \frac{L(r)}{L(\Delta + r)} + \frac{4L(r)}{L\left(\sqrt{\Delta^2 + r^2}\right)}\right) \tag{7}$$

where ζ is the *Riemann zeta function*. In [8, Eq. (10)] another approximation is proposed[1], that is

$$f^{\mathrm{c}}(r) = \frac{2r^\eta}{R^2(\eta - 2)(\Delta - r)^{\eta - 2}} \tag{8}$$

where R is given by (4). (The approximation proposed in [5] is too loose since it considers only the six neighboring base stations.)

The following example compares the approximations of $\bar{f}(r)$ described above.

Example 6: We represent the functions $\bar{f}(r)$, $f^{\mathrm{a}}\left(re^{i\pi/12}\right)$, $f^{\mathrm{b}}(r)$ and $f^{\mathrm{c}}(r)$ of the distance r for $\eta = 3, 4, 5$ on Fig. 8. We observe that the approximation f^{a} is good where as the approximation f^{b} and f^{c} may be far from the exact value $\bar{f}(r)$. This is particularly true for f^{c} when $\eta \geq 4$.

A. Discussion

Note that the calculations of the reference value $\bar{f}(r)$ with the help of (3) (or the approximation $f^{\mathrm{a}}\left(re^{i\pi/12}\right)$) are not too time consuming. Indeed, the number of terms to be calculated is not too large as shown in Example 1. Thus we recommend to use these methods if one seeks for good precision. This is particularly the case for operational use as for example dimensioning a real network. Nevertheless when only a first rough result is desired, we may also use the approximation $f^{\mathrm{b}}(r)$ given by (7) which overestimates the f-factor, thus leading to a *safe* dimensioning (i.e., the QoS really perceived by the users would be better than the target value used in dimensioning). Finally, note that the use of $f^{\mathrm{c}}(r)$ given by (8) when $\eta \geq 4$ leads to an underestimation of the f-factor by more than 30% which may lead to unsafe dimensioning (i.e., the QoS really perceived by the users may be much worst than the target value used in dimensioning).

[1]Note that the quantity calculated in [7, Eq. (II.7)] is $\sum_{v \neq u} \frac{L_{v,m}}{L_{u,m}}$ (for $m \in u$) which is not the f-factor.

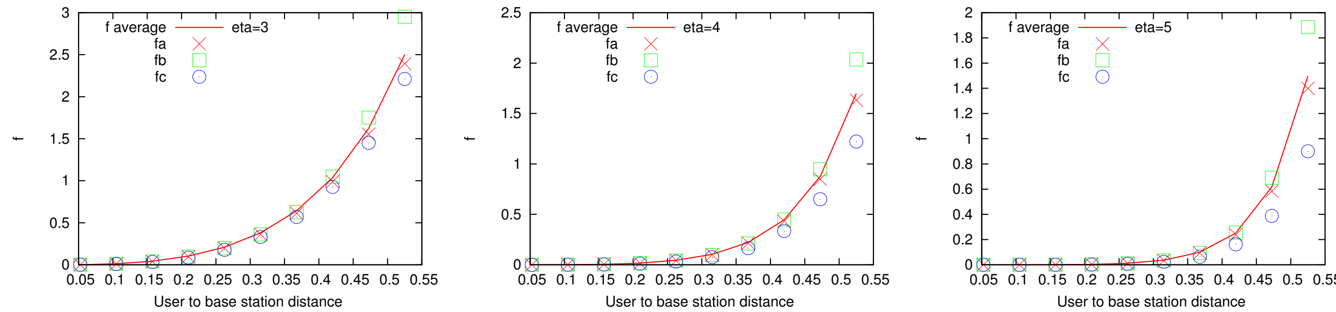

Fig. 8. The functions $\bar{f}(r)$, $f^{\mathrm{a}}\left(re^{i\pi/12}\right)$, $f^{\mathrm{b}}(r)$ and $f^{\mathrm{c}}(r)$ of the distance r for $\eta = 3, 4, 5$ from left to right respectively.

B. Sectorial antennas

The f-factor with sectorial antennas was first investigated in [3], [13] which are the basis of [6, §13.C]. The main result is that the f-factor with S-sectorial antennas, denoted by $f^{(S)}(m)$, is expressed in terms of the one with omnidirectional antennas, denoted by $f(m)$, and the radiation pattern of the sectorial antennas $G(\theta)$. Assume that there are S sectors (typically $S = 3$) which we index by $s = 0, 1, \ldots, S-1$. For a location m in sector 0, we have from [6, End of p.222]

$$f^{(S)}(m) \simeq \frac{\sum_{s=1}^{S-1} G(\theta - s2\pi/S)}{G(\theta)} + \frac{\bar{G}}{G(\theta)} f(m) \qquad (9)$$

where $\theta = \arg(m)$ and $\bar{G} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sum_{s=0}^{S-1} G(\theta - s2\pi/S)\, d\theta$. (The formula in [8, Equation (21)] is a particular case of the above one for $S = 3$.) For more details and in particular for expressions of the f-factor moments see [6, §13.C].

C. Comparison to measurements

Measurements on the field as well as results of calculations of the f-factor for an irregular network (with 3-sectorial antennas) are reported in [10]. It is shown in particular that there is a good fit between the f-factor mean deduced from measurements and that from calculations. The obtained value is $\bar{f}^{(3)} \simeq 0.6$.

Unfortunately the value of η and the antenna radiation pattern are not given in [10]. Thus we take the typical value in urban regions $\eta = 3.38$ and consider a sectorial antenna with perfect radiation pattern (in which case $\bar{f}^{(3)} = \bar{f}$). Using Equation (5) we get $\bar{f}^{(3)} = \bar{f} \simeq 0.7$.

As regards to the variations of the f-factor with distance, the simulations reported in [10] give an f-factor varying between 0.02 at cell center and 1.7 at cell edge. If we apply (2) with $\eta = 3.38$ we get an f-factor varying between 0 at cell center and 2 at cell edge. This shows a relatively good agreement between the results of our calculus and those in [10]; despite the irregularity of the considered network there and the lack of information about the value of η and the antenna pattern.

V. Conclusion

We build a method to calculate the f-factor with a desired precision. Taking a small value of the precision permits to get a *reference* value for the f-factor. Calculating the f-factor with this reference method, we observe numerically that the f-factor is: (1) increasing with the distance between the user and its serving base station; and (2) slowly varying with the angle when the distance is fixed. Moreover, we establish good approximations of the f-factor average over the angle when the distance is fixed, and over the cell.

Previously proposed approximations for the f-factor as function of distance as well as a new approximation are compared to the reference value. It is shown that the previous approximations are not always close to the reference. One should choose the approximation carefully since the performance of cellular networks depend strongly on the f-factor. The results in our paper help to make the appropriate choice. In particular, since the calculations of the f-factor reference value (or the approximation proposed in the present paper) are not too time consuming, we recommend to use these methods if one seeks for good precision. This is particularly the case for operational use as for example dimensioning a real network.

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