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Analytical expressions for blocking and dropping probabilities for mobile streaming users in wireless cellular networks

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Abstract We consider a wireless cellular network serving *streaming* traffic. We study in this context the effect of the users mobility on their *quality of service* (QoS). If the arrival of a new user violates the capacity constraint, then his call is *blocked*. If the user is first admitted but the capacity constraint is violated later when he attempts to move, then his call is *dropped*. The *blocking* and *dropping probabilities* are the main QoS indicators in this model called *forced termination* (FT). We introduce an alternative model, called *transitions backtrack* (TB), where a user backtracks when his motion violates the capacity constraint. In this model, a relevant QoS indicator is the number of times the user backtracks called *number of motion blocking per call*. We propose some *explicit expressions* for the above QoS indicators as functions of the mean user speed. These expressions are validated by simulations. In particular we observe that the dropping probability in the FT model is well approximated by the number of motion blocking per call in the TB model which is expressed analytically.

Keywords Communication system performance · Mobility · Dropping probability · Markov processes · Counting measures · Point processes

1 Introduction

We consider a wireless cellular network serving *streaming* (i.e., real-time; such as voice, video streaming, etc.) users who require a predefined transmission rate for some duration. If a new call arrival violates the capacity constraint, then the call is *blocked*. If the user is first admitted but the capacity constraint is violated later when he attempts to move, then his call is *dropped*. The fractions of blocked and dropped calls in the long run of the system, called respectively, *blocking* and *dropping probabilities*, are the main indicators of the quality of service (QoS) perceived by the users. Analytical evaluation of these QoS indicators is crucial for the network *dimensioning*; i.e., evaluating the minimal number of base stations assuring some QoS (for some given traffic demand). This permits to minimize the network cost.

In real-life networks, the blocking and dropping probabilities should be maintained less than 10^{-2} and 10^{-3} respectively. The dropping probability may sometimes be the limiting factor when dimensioning the network. But the lack of an efficient method to evaluate it led the engineers to consider only the blocking probability which may lead to an unsafe dimensioning (i.e., the dropping perceived by the users may be much larger than 10^{-3}).

Unfortunately, the evaluation of such QoS parameter is a hard task. Nevertheless it may be decomposed into three subproblems. **First** *information theory* characterizes the performance of each single link. We will suppose that such a characterization is given. **Secondly** we should take into account the interference between the different links which depends on the relative geographic positions of the users. This interaction between all the users should be taken into account when *resource* (power and bandwidth) is *allocated* to users. Some resource allocations are *opportunistic*; i.e., allocate the resource to the user having the most favorable channel state. **Finally**, the users arrivals, mobility and departures modify their mutual interference which impacts the blocking and dropping probabilities in the long run of

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the network. This subproblem may be studied by using tools from *queueing theory*.

In reassembling the solutions of the above subproblems, we have to consider the separation of their *time scales*. A reasonable approach is to assume that the above three subproblems; i.e., information theory, resource allocation and queueing theory; have respectively increasing time-scales. In other words, information theory corresponds to the fastest time-scale whereas queueing theory corresponds to the slowest one.[1]

Indeed, users mobility has effects on the above three subproblems. Its impact on the information theory subproblem and on the possibility of an opportunistic resource allocation has already been investigated, at least qualitatively in previous studies (see for example [16, Chap. 9] and [24, §6.2]).

This paper focuses on the users mobility effect on QoS for the queueing theory subproblem. To do so, we assume a given characterization of the single link performance by information theory and a given resource allocation scheme which do not vary with user's speed. Moreover, we will suppose that the users mobility does not modify the order of the time-scales of the three above sub-problems. Despite these simplifying assumptions, the problem at hand remains practically important and difficult to solve. Its study will give a useful comprehension of the effect of mobility on QoS.

1.1 Brief description of our approach

We first consider a model without capacity constraint. This *free* model may be seen as corresponding to a theoretical network with infinite resources. In order to take into account the limitation of the resources in real-life networks, the dynamics of the *free* model are modified each time an arrival or displacement leads to a violation of the capacity constraint. New arrivals in such situations are blocked.

As far as displacement is concerned, we consider two possible modifications of the free process, which lead to *two different loss models*.

- The *transitions backtrack model* (TB), assumes that the moving user in question immediately backtracks to his previous location.
- The other one, called *forced termination model* (FT), assumes that the moving call is dropped (i.e.; prematurely interrupted).

The models TB and FT are obviously equivalent when there is no user mobility. The TB model is more natural if users backtrack when the quality of their calls becomes unacceptable. It will be mathematically analyzed. The FT model (at a non-null speed) will be simulated only since it is analytically less tractable. In both models we observe the effect of user mobility on the QoS.

More precisely, in the TB model, we establish analytical expressions for the *(access) blocking probability* that under some quite natural assumptions does not depend on the mean user speed. In this model we also analyze the *mean number of blocked motions per call* (i.e., number of times the user backtracks).

For the FT model, in which, besides call (access) blocking, one observes call dropping, we show that the *dropping probability* is well approximated by the mean number of blocked motions per call evaluated for the corresponding TB model. We validate this approximation studying Orthogonal Frequency-Division Multiple Access (OFDMA) networks. The main results of the present paper are:

1. Analytic expressions of the mean number of blocked motions per call in the TB model [see for example (5)].
2. Explicit approximations of the dropping probability in the FT model [see for example (7)].
3. A simple bound for the blocking probability in the FT model [see (8)].

1.2 Paper organization

The remaining part of this paper is organized as follows. In the following subsection we briefly review the related work. Section 2 introduces the basic model and tools. In Sect. 3 we describe and analyze two loss models and establish our explicit approximations of the QoS indicators. In Sect. 4 we develop our main numerical example—analysis of the streaming in OFDMA networks. In "Appendix" we describe a particular model for mobility called *completely aimless mobility*. "Appendix" contains some more technical elements of our mathematical analysis.

1.3 Related work

Many studies of QoS in cellular networks do not take into account the users mobility ([1, 14, 25]). In such context the dropping probability may not be defined. The papers [17, 26, 29] consider users mobility. In [26] and [17] the dropping probability is evaluated considering a "phantom" mobile moving in the network, which does not affect its state. In [29], explicit expressions for the blocking and dropping probabilities are given for two limiting regimes: no mobility and infinite mobility. Approximations are also given for intermediate mobility regimes. These approximations are based on the study of some special and rather

[1] The reader may refer to [8, §II.A] for more discussion.

simplistic network architectures (e.g. cells located on a ring or on the line). In [26] Erlang fixed-point approximations are proposed to calculate the blocking and dropping probabilities.

In [9] upper bounds for the blocking and 'outage' probabilities are derived under a certain monotonicity property. The paper [32] shows by simulations that mobility has an important impact on the blocking and dropping probabilities. It is observed in particular that blocking probability decreases whereas dropping probability increases as mobility increases, but no analytical expressions for these parameters are given.

More recently, the authors of [31] propose an analytical model to calculate the blocking and 'dropping' probabilities for an integrated cellular Ad hoc relaying system. But the dropping there is not related to geographical mobility of the users, but to the transfer of mobiles between the cellular and the Ad hoc system. In [15] the effect of heterogeneous mobile terminals on the blocking and dropping probabilities is investigated by simulations.

The authors of [13] propose to approximate the FT model (with simple capacity constraints) with the TB one. Moreover, they establish an upper bound of the error induced on the *throughput* by such approximation. Unfortunately, the blocking and the dropping probabilities are not studied there. This is fulfilled in the recent paper [10] where it is proposed to approximate the FT model by the so-called *redial rate approximation* which is different from the TB model. Indeed, the TB model is interesting in it self for the situations where (pedestrian) users backtrack when the quality of their calls becomes unacceptable. Moreover, comparing the blocking (and the dropping) probabilities of the FT and TB models will give an interesting insight on these parameters.

It is shown in [2, 3] that in some cases the network may oscillate between some states with a significant sojourn time in each state (a phenomenon called *metastability*). In the present work we shall consider the average of the blocking and dropping probabilities over time intervals sufficiently large so that the ergodic averages over the different states are obtained.

The present paper relies on and continues the work in [6, 19] where we studied Code-Division Multiple Access (CDMA) networks.[2] Recent cellular networks use OFDMA.[3] For such networks, we will consider that the *capacity constraint* is that the resource (power and bandwidth) allocation is feasible as proposed in [7] (more details will be given in Sect. 4.1). Since the capacity constraints in OFDMA networks are different from those in CDMA ones, our previous results has to be extended and checked again for OFDMA. In doing so, we present in more details the results of [6], discuss more deeply some recent related works and give some interpretations of the established relations.

The effect of mobility on *elastic* traffic (i.e., non-real-time; such as web browsing, mail, etc.) is studied separately in [21]. The two studies have some common basis, in particular the *completely aimless mobility* model described in "Appendix". Note that for the queueing theory sub-problem, it is shown in [21] that mobility ameliorates the QoS (increases throughput) for elastic calls. We will investigate in the present paper whether mobility ameliorates or degrades the QoS (blocking, dropping) for streaming calls.

In [22] we study a network serving simultaneously streaming and elastic traffic users who don't move during their calls. The following observation made in the previous reference remains also true in the case of mobility. If streaming traffic has preemptive priority over elastic traffic, then the evolution of the streaming users is *independent* of the elastic ones; in particular, the *blocking and dropping probabilities* of streaming calls is the same as if there were no elastic ones.

2 Basic model and tools

2.1 System state and its evolution

We will consider a wireless network composed of a finite set of base stations (BS). We assume that each BS u serves the users located in a given geographic region called *cell*. We assume that the cells are bounded subsets of $\mathbb{R}^2$. With a slight abuse of notation, we will use the same letter for the BS and its cell. In particular, $x \in u$ means that the location x is served by BS u.

Let $\mathbb{D}$ be the union of all the cells. Elements $x \in \mathbb{D}$ denote *geographic locations of users* in the system. Configurations $\{x_i\} \subset \mathbb{D}$ of users in the system are identified by corresponding counting measures $\nu = \sum_i \varepsilon_{x_i}$; where the Dirac measure ε_x is defined by $\varepsilon_x(A) = 1$ if $x \in A$ and 0 otherwise, consequently $\nu(A)$ is the number of users in the set $A \subset \mathbb{D}$. We denote by $\mathbb{M}$ the set of *all finite configurations of users* (i.e., finite counting measures) on $\mathbb{D}$.

We will describe the temporal evolution of the configuration of users in $\mathbb{D}$ by a continuous-time jump Markov process, which takes values in $\mathbb{M}$. This process evolves because of users arriving, moving or leaving the system, with only one such event being possible at a time.

[2] A typical example of CDMA system is the Universal Mobile Telecommunications System (UMTS).

[3] Typical examples of OFDMA systems are the 3GPP Long Term Evolution (LTE) system and IEEE 802.16 WirelessMAN Air Interface standard (WiMAX).

2.2 Free (MPL) process

The process describing the evolution of the streaming users in the *absence of any capacity constraint* is called *free process*. A natural candidate in the context of wireless communications is the *Markov Poisson Location* (MPL) process where users arrive, move and leave the system independently of each other. Here we describe its dynamics more precisely. It will be used in our analysis of OFDMA in Sect. 4.

2.2.1 Arrivals, call durations and bit-rates

For a given subset $A \subset \mathbb{D}$ inter-arrival times of users to A are independent of everything else, exponential random variables with mean $1/\lambda(A)$, where $\lambda(\cdot)$ is some given *intensity measure of arrivals* to $\mathbb{D}$ per unit of time. In homogeneous traffic conditions, we take $\lambda(dx) = \lambda\ dx$, where λ is the mean number of arrivals per unit of area and per unit of time. We always assume $\lambda(\mathbb{D}) < \infty$ (thus, in homogeneous case, $\mathbb{D}$ has a finite area).

We assume that each user has a call with *exponentially distributed duration* of parameter $\mu > 0$.[4] (The exponential assumption can be relaxed in the subsequent analysis of the transitions backtrack in the MPL model due to the so-called insensitivity property [4, p. 123].) Each user requires some given transmission bit-rate.

2.2.2 Mobility

Assume that the users move independently of each other in $\mathbb{D}$.[5] The *sojourn duration* of a given user at a location $x \in \mathbb{D}$ is assumed independent of everything else and *exponentially distributed* (see [12]) *with parameter* $\lambda'(x)$. Each user finishing its sojourn at location *x moves to a new region dy* according to some *probability kernel* $p'(x, dy)$. The individual user mobility may then be described by a Markov process on $\mathbb{D}$ with the following generator $\lambda(x, dy) = \lambda'(x)p'(x, dy)$ called *mobility kernel*. We assume that it admits a stationary distribution which we denote by $\sigma(\cdot)$ and call *mobility stationary distribution*; it satisfies the following equations:

$$\begin{gathered} \sigma(\mathbb{D}) = 1 \\ \int_A \lambda(x, \mathbb{D})\sigma(dx) = \int_{\mathbb{D}} \lambda(x, A)\sigma(dx), \quad A \subset \mathbb{D}. \end{gathered} \tag{1}$$

We give in "Appendix" the explicit expressions for the mobility kernel for a particular example of mobility model: the so-called *completely aimless mobility*. Note that in this particular case the mobility kernel is proportional to the *average speed of users* denoted υ. We will always assume such proportionality. It follows in particular that the mobility stationary distribution is independent of υ.

2.2.3 Generator

We consider now the process describing the temporal evolution of the configuration of users in $\mathbb{D}$ due to arrivals, mobility and departures. In order to get its generator we use the following reasoning (which may be more formalized by using [11, Ch. 9]). Assume that the system is in some state $\nu \in \mathbb{M}$. An arrival of a user at position y brings the system to state $\nu + \varepsilon_y$, and this occurs with rate $\lambda(dy)$. A departure of a user from position x brings the system to state $\nu - \varepsilon_x$, and this occurs with rate $\mu\nu(dx)$. Finally a transition of a user from position x to position y brings the system to state $\nu - \varepsilon_x + \varepsilon_y$, and this occurs with rate $\lambda(x, dy)\nu(dx)$. Thus the generator q of the *Markov Poisson Location* (MPL) process is: for $\nu \in \mathbb{M}, \Gamma \subset \mathbb{M}$

$$\begin{aligned} q(\nu, \Gamma) = &\int_{\mathbb{D}} 1(\nu + \varepsilon_y \in \Gamma)\lambda(dy) \\ &+ \int_{\mathbb{D}} 1(\nu - \varepsilon_x \in \Gamma)\mu\nu(dx) \\ &+ \int_{\mathbb{D}\times\mathbb{D}} 1(\nu - \varepsilon_x + \varepsilon_y \in \Gamma)\lambda(x, \mathrm{d}y)\nu(dx) \end{aligned}$$

corresponding to arrivals, departures and mobility of users respectively.

2.2.4 Stationary distribution

We introduce a " virtual" location $o \notin \mathbb{D}$ which represents the initial location of calls arriving to or leaving the system. We extend the kernel $\lambda(x, dy)$ to $\overline{\mathbb{D}} = \mathbb{D} \cup \{o\}$ by defining the rates from o and to o as the arrival intensity $\lambda(\cdot)$ and the call duration parameter μ respectively; i.e.

$$\lambda(o, A) = \lambda(A), \quad \lambda(x, \{o\}) = \mu, \quad x \in \mathbb{D}, A \subset \mathbb{D}.$$

We call this extended kernel the *traffic kernel*. We assume that it admits an invariant measure $\rho(\cdot)$ satisfying the following equations

$$\rho(o) = 1, \quad \int_A \lambda(x, \overline{\mathbb{D}})\rho(dx) = \int_{\overline{\mathbb{D}}} \lambda(x, A)\rho(dx), \quad A \subset \overline{\mathbb{D}},$$

(called *traffic equations*) and $\rho(\mathbb{D}) < \infty$. We call $\rho(\cdot)$ the *traffic intensity*.

Note for future reference that the above equations are equivalent to

$$\begin{aligned} &\lambda(\mathbb{D}) = \mu\rho(\mathbb{D}) \\ &\int_A \lambda(x, \mathbb{D})\rho(dx) + \mu\rho(A) = \int_{\mathbb{D}} \lambda(x, A)\rho(dx) \\ &\quad + \lambda(A), \quad A \subset \mathbb{D} \end{aligned} \tag{2}$$

[4] In the case of multi-class users, $\lambda(\cdot)$, μ can depend on the user class.

[5] In the case of multi-class users we assume that the users do not change their classes during the service.

Under the above assumptions, the MPL process admits as stationary distribution Π the distribution of a Poisson point process on $\mathbb{D}$ of intensity $\rho(\cdot)$.[6] (This is a classical result in queueing theory when $\mathbb{D}$ is discrete, for a detailed proof when $\mathbb{D}$ is continuous see for example [19, Proposition 28]).

It is useful to know whether Π depends or not on the average user speed υ.

Proposition 1 *If the arrival intensity $\lambda(\cdot)$ is proportional to the mobility stationary distribution $\sigma(\cdot)$, then the traffic intensity is equal to*

$$\rho(\cdot) = \sigma(\cdot)\lambda(\mathbb{D})/\mu. \tag{3}$$

Moreover, the stationary distribution Π of the free process does not depend on the average user speed υ.

Proof If $\lambda(\cdot)$ is proportional to $\sigma(\cdot)$, then $\sigma(\cdot) = \lambda(\cdot)/\lambda(\mathbb{D})$. Let $\rho(\cdot) = \sigma(\cdot)\lambda(\mathbb{D})/\mu$. It is easy to check that the first equation in (2) holds true. On the other hand, for all $A \subset \mathbb{D}$,

$$\begin{aligned}
&\int_A \lambda(x,\mathbb{D})\rho(dx) + \mu\rho(A) \\
&\quad = \int_A \lambda(x,\mathbb{D})\sigma(dx)\frac{\lambda(\mathbb{D})}{\mu} + \lambda(\mathbb{D})\sigma(A) \\
&\quad = \int_{\mathbb{D}} \lambda(x,A)\sigma(dx)\frac{\lambda(\mathbb{D})}{\mu} + \lambda(\mathbb{D})\sigma(A) \\
&\quad = \int_{\mathbb{D}} \lambda(x,A)\rho(dx) + \lambda(A)
\end{aligned}$$

where for the second equation we use (1). Thus the second equation in (2) holds also true. This proves that $\rho(\cdot)$ given by (3) satisfies (2). Recall that $\sigma(\cdot)$ is invariant with respect to υ, then $\rho(\cdot)$ is invariant too. Since Π is the distribution of a Poisson point process on $\mathbb{D}$ of intensity $\rho(\cdot)$, Π does not depend on υ. $\square$

2.3 Modeling losses

Consider a Markov process describing a free evolution of the system, e.g. the MPL process described in the previous section. Suppose that the " true" evolution of the system is subject to some constraints, which can be expressed as the limitation of the original state space $\mathbb{M}$ of all configurations of users, to a given fixed subset $\mathbb{M}^{\mathrm{f}} \subset \mathbb{M}$ of *feasible configurations*. (We may consider $\mathbb{M} \setminus \mathbb{M}^{\mathrm{f}}$ as corresponding to violation of the capacity constraint.) We will always assume that $\mathbb{M}^{\mathrm{f}}$ has the following *monotonicity property:* if a configuration v is feasible ($v \in \mathbb{M}^{\mathrm{f}}$) then any subset $v' \subset v$ is also feasible, i.e.; $v' \in \mathbb{M}^{\mathrm{f}}$.

[6] In the case of multi-class users, who do not change class during their calls, we assume that the same assumption holds true for each class separately and Poisson point processes describing users of different classes are independent.

Examples of $\mathbb{M}^{\mathrm{f}}$ useful in modeling of wireless cellular networks are presented in Sect. 4.1. We remark here only that in general, the feasibility condition (i.e.; the condition for $v \in \mathbb{M}^{\mathrm{f}}$) depends on individual user locations. Note also that the monotonicity property of $\mathbb{M}^{\mathrm{f}}$ implies that all user departures preserve the feasibility of configurations.

We assume that *the "true" system with losses, started at an initial state in $\mathbb{M}^{\mathrm{f}}$ follows the same dynamic as the free process as long as it stays in $\mathbb{M}^{\mathrm{f}}$ and is forced to modify its behavior each time an attempt of a transition from $\mathbb{M}^{\mathrm{f}}$ to $\mathbb{M} \setminus \mathbb{M}^{\mathrm{f}}$ occurs.* In the next section we will consider two possible modifications applied at such epochs. They lead to two different models, one of which is analytically tractable, the other one is more difficult to analyze. Studying both of them we will be able to propose some explicit formulae for QoS.

3 Two loss models

In this section we describe and analyze two different modifications of the free process dynamics making it stay in the set $\mathbb{M}^{\mathrm{f}}$ of feasible configurations. For simplicity we restrict ourselves to MPL process as the free process and assume that the latter is *reversible* and *ergodic*. (Some results may be extended to more general scenarios).

3.1 Transitions backtrack (TB) model

In this model the free process is modified to stay in $\mathbb{M}^{\mathrm{f}}$ by applying transitions backtrack for the arrivals as well as for displacements. More precisely, the following rules are applied when an attempt of a transition from $\mathbb{M}^{\mathrm{f}}$ to $\mathbb{M} \setminus \mathbb{M}^{\mathrm{f}}$ occurs.

- (TB1) Any call arrival that would result in taking the process to a state outside $\mathbb{M}^{\mathrm{f}}$ is not allowed to enter to the system (blocked) and excluded from its further evolution.
- (TB2) Any displacement of a user in the system that would take the process to a state outside $\mathbb{M}^{\mathrm{f}}$ is ignored. This means that the user in question is instantaneously taken back to his previous location, and the system keeps on evolving with this user according to the free dynamics until the next attempt to leave $\mathbb{M}^{\mathrm{f}}$. (This model is suitable if a user backtracks when the quality of his call becomes unacceptable.)

The above modification of the free process evolution corresponds to the so-called truncation of q to $\mathbb{M}^{\mathrm{f}}$. This means that the generator q^{tb} of the TB process is given by

$$q^{\mathrm{tb}}(v,\Gamma) = \begin{cases} q(v,\Gamma \cap \mathbb{M}^{\mathrm{f}}) & \text{if } v \in \mathbb{M}^{\mathrm{f}}, \Gamma \subset \mathbb{M} \\ q(v,\Gamma) & \text{if } v \in \mathbb{M} \setminus \mathbb{M}^{\mathrm{f}}, \Gamma \subset \mathbb{M} \end{cases}$$

3.1.1 Stationary distribution

Let us denote by Π^{tb} the stationary distribution of the TB process. We have the following result making the TB model more tractable.

Claim The TB of the MPL model leads to a reversible and ergodic process, whose stationary distribution Π^{tb} is equal to the truncation of Π to $\mathbb{M}^f$; i.e.; for any $\Gamma \subset \mathbb{M}, \Pi^{tb}(\Gamma) = \Pi(\Gamma \cap \mathbb{M}^f)/\Pi(\mathbb{M}^f)$.

Proof Since the MPL (free) process is reversible with respect to its stationary distribution Π, the result follows from [28, Proposition 3.14]. □

3.1.2 Access and motion blocking probabilities

We define the (access) blocking probability in the TB model by the following ergodic limit

$$b^{tb} = \lim_{t\to\infty} \frac{\#\{\text{blocked arrivals in } [0,\mathrm{t}]\}}{\#\{\text{all arrivals in } [0,t]\}}.$$

where # denotes the cardinality and [0, t] designates a time interval. One can also consider the blocking probability related to mobility inside the system, i.e.; the ratio of the number of blocked displacements with respect to the number of all displacements of users in the system. However another characteristic shall be more useful in our analysis. It is the *mean number of motion blocking per call*

$$d^{tb} = \lim_{t\to\infty} \frac{\#\{\text{blocked displacements in } [0,\mathrm{t}]\}}{\#\{\text{non-blocked arrivals in } [0,t]\}}.$$

Note that d^{tb}can be larger than 1.

Both b^{tb} and d^{tb} admit some more explicit expressions in terms of the stationary distribution Π^{tb}. In particular, the following formula can be seen as a spatial extension of the well-known Erlang formula. It follows from Proposition B.4.

Proposition 2 *The blocking probability is given by*

$$b^{tb} = \int_{\mathbb{D}} p^{tb}(y)\lambda(dy)/\lambda(\mathbb{D})$$

where

$$\begin{aligned} p^{tb}(y) &= \Pi^{tb}\{v + \varepsilon_y \notin \mathbb{M}^f\} \\ &= \Pi\{v \in \mathbb{M}^f : v + \varepsilon_y \notin \mathbb{M}^f\}/\Pi(\mathbb{M}^f) \end{aligned}$$

The analogy to the Erlang formula consists in expressing the intensity p^{tb} (y) of blocking of users arriving at $y \in \mathbb{D}$ by the *conditional probability that the stationary configuration of users in the free (here Poisson) process cannot admit a new user at y given the configuration is in* $\mathbb{M}^f$. We will describe some practical methods to evaluate the blocking probability in Sect. 4.2 below.

Remark 1 Note by the above proposition and Proposition 1 that if the arrival intensity is proportional to the mobility stationary distribution, then *the blocking probability* b^{tb} *does not depend on the mean user speed* υ.

The following proposition gives the expression of the mean number of motion blocking per call. It follows from Proposition B.5.

Proposition 3 *The mean number of motion blocking per call is given by*

$$d^{tb} = \frac{\int_{\mathbb{D}\times\mathbb{D}} \Pi^{tb}\{v + \varepsilon_x \in \mathbb{M}^f, v + \varepsilon_y \notin \mathbb{M}^f\}\lambda(x, dy)\rho(dx)}{\mu \int_{\mathbb{D}}(1 - p^{tb}(x))\rho(dx)}. \tag{4}$$

Proof We use Proposition B.5. Recall that in the case of the MPL free process Π is the distribution of the Poisson point process with intensity ρ, and Π^{tb} is the truncation of Π to $\mathbb{M}^f$. Thus, the denominator in the formula for d^{tb} given in Proposition B.5 is equal to

$$\begin{aligned} \mu E_{\Pi^{tb}}[v(\mathbb{D})] &= (\Pi(\mathbb{M}^f))^{-1}\mu \mathbf{E}_\Pi\left[\int_{\mathbb{D}} 1(v \in \mathbb{M}^f)v(dx)\right] \\ &= (\Pi(\mathbb{M}^f))^{-1}\mu \int_{\mathbb{D}} \mathbf{E}_\Pi[1(v + \varepsilon_x \in \mathbb{M}^f)]\rho(dx) \\ &= \mu \int_{\mathbb{D}} (1 - p^{tb}(x))\rho(dx), \end{aligned}$$

where the last but one equality follows from the Campbell formula [5]. Similarly one can show using Campbell formula that

$$\begin{aligned} &E_{\Pi^{tb}}[1(v - \varepsilon_x + \varepsilon_y \notin \mathbb{M}^f)v(dx)] \\ &= \Pi(\mathbb{M}^f)^{-1}\Pi\big(v + \varepsilon_x \in \mathbb{M}^f, v + \varepsilon_y \notin \mathbb{M}^f\big)\rho(dx). \end{aligned}$$

Applying the above expression to the numerator in the formula for d^{tb} given in Proposition B.5 one concludes the proof. □

Note that d^{tb} depends on υ even if Π is invariant with respect to υ. In fact, d^{tb} increases linearly in υ as a consequence of the linear dependence of $\lambda(x, dy)$ on υ.

In order to calculate the value $\Pi^{tb}\{\ldots\}$ in (5), we decompose it as follows

$$\begin{aligned} &\Pi^{tb}\{v + \varepsilon_x \in \mathbb{M}^f, v + \varepsilon_y \notin \mathbb{M}^f\} \\ &= \Pi^{tb}\{v + \varepsilon_x \in \mathbb{M}^f\}\Pi^{tb}\{v + \varepsilon_y \notin \mathbb{M}^f | v + \varepsilon_x \in \mathbb{M}^f\} \\ &= p^{tb}(x)\Pi^{tb}\{v + \varepsilon_y \notin \mathbb{M}^f | v + \varepsilon_x \in \mathbb{M}^f\} \\ &= p^{tb}(x)\Pi\{v + \varepsilon_y \notin \mathbb{M}^f | v \in \mathbb{M}^f - \varepsilon_x\} \end{aligned}$$

Note that the term $\Pi\{v + \varepsilon_y \notin \mathbb{M}^f | v \in \mathbb{M}^f - \varepsilon_x\}$ in the above equation may be viewed as an *access blocking probability* with respect to the modified feasibility set $\mathbb{M}^f - \varepsilon_x$. Therefore it may be evaluated using the access blocking probability calculation methods (see Sect. 4.2 below).

In some cases d^{tb} can be evaluated more explicitly as shown in the following propositions.

Proposition 4 *If*

$$\Pi^{\mathrm{tb}}\{v+\varepsilon_x \in \mathbb{M}^{\mathrm{f}}, v+\varepsilon_{\frown} \notin \mathbb{M}^{\mathrm{f}}\} = \Pi^{\mathrm{tb}}\{v+\varepsilon_x \in \mathbb{M}^{\mathrm{f}}\}\Pi^{\mathrm{tb}}\{v+\varepsilon_y \notin \mathbb{M}^{\mathrm{f}}\}$$

(which is the case e.g. when $\mathbb{M}^{\mathrm{f}}$ is in the form (12) below) then

$$d^{\mathrm{tb}} = \frac{\int_{\mathbb{D}\times\mathbb{D}} p^{\mathrm{tb}}(y)(1-p^{\mathrm{tb}}(x))\lambda(x,dy)\rho(dx)}{\mu\int_{\mathbb{D}}(1-p^{\mathrm{tb}}(x))\rho(dx)}.$$

Proof *Immediate from Propositions 2 and 3.* □

Proposition 5 *Besides the assumption in Proposition 4, assume that each location $x \in \mathbb{D}$ plays the same role. Then*

$$d^{\mathrm{tb}} = \frac{\lambda(\mathbb{D})}{\mu} b^{\mathrm{tb}}$$

where $\lambda(\mathbb{D}) \equiv \lambda(x,\mathbb{D})$ (which is independent of $x \in \mathbb{D}$). In the particular case of the mobility kernel (18)[7]

$$d^{\mathrm{tb}} = \frac{2\upsilon}{\pi R\mu} b^{\mathrm{tb}} \tag{5}$$

where υ is the user's average speed and R is the cell radius.

Proof Since each location $x \in \mathbb{D}$ plays the same role, (1) the function $p^{\mathrm{tb}}(x) \equiv b^{\mathrm{tb}}$ is constant over $\mathbb{D}$; (2) the measure $\rho(\cdot)$ is uniform; and (3) the measure $\lambda(x,\cdot) \equiv \lambda(\cdot)$ is independent of $x \in \mathbb{D}$. We deduce from Proposition 4 that

$$d^{\mathrm{tb}} = \frac{\int_{\mathbb{D}\times\mathbb{D}} b^{\mathrm{tb}}\lambda(dy)\rho(dx)}{\mu\int_{\mathbb{D}}\rho(dx)} = \frac{\lambda(\mathbb{D})}{\mu} b^{\mathrm{tb}}$$

In the particular case of the mobility kernel (18) we have $\lambda(\mathbb{D}) = 2\upsilon/(\pi R)$ which substituted in the above display implies (6). □

We give now an interpretation of Eq. 5. Observe firstly that the average sojourn duration of a user within a cell is $\pi R/(2\upsilon)$ [this may be deduced from (17)]. Since the mean call duration is $1/\mu$, we deduce that the mean number of cells crossed by a call equals

$$\frac{1/\mu}{\pi R/(2\upsilon)} = \frac{2\upsilon}{\pi R\mu}$$

Equation 5 reads as follows: the mean number of motion blocking per call equals the number of cells crossed by a call multiplied by the probability of access blocking. This can be interpreted by saying that each time a user attempts to move towards another cell, his movement is blocked with a probability equal to the probability of blocking a new call access.

[7] Which corresponds to *completely aimless intercell* mobility.

3.2 Forced termination (FT) model

In this model the dynamics of the free process are modified according to the following rules when an attempt of a transition from $\mathbb{M}^{\mathrm{f}}$ to $\mathbb{M}\setminus\mathbb{M}^{\mathrm{f}}$ occurs.

(FT1) Any call arrival that would result in taking the process to a state outside $\mathbb{M}^{\mathrm{f}}$ is not allowed to enter to the system and excluded (blocked) from its further evolution.

(FT2) Any displacement of a user in the system that would take the process to a state outside $\mathbb{M}^{\mathrm{f}}$ leads to the forced termination (dropping) of the call of this user, i.e.; rejection of this user from the system and from its further evolution.

Each time the rule (FT1) or (FT2) is applied we say that the corresponding user (call) is lost.

3.2.1 Blocking and dropping probabilities

Assume that the FT process is ergodic and let Π^{ft} be its stationary distribution. The main QoS indicators of the FT model are blocking and dropping probabilities defined respectively by the following ergodic limits

$$b^{\mathrm{ft}} = \lim_{t\to\infty} \frac{\#\{\text{blocked arrivals in } [0,\mathrm{t}]\}}{\#\{\text{all arrivals in } [0,t]\}}$$

$$d^{\mathrm{ft}} = \lim_{t\to\infty} \frac{\#\{\text{dropped calls in } [0,\mathrm{t}]\}}{\#\{\text{non-blocked arrivals in } [0,t]\}}$$

Unfortunately neither Π^{ft} nor the FT blocking and dropping probabilities can be expressed analytically.

3.3 Approximations

In this section we propose some approximation of the FT dropping probability and some upper bound for the FT blocking probability, which can be calculated using the TB model. This approximation and bound are heuristic, but will be validated by simulations in Sect. 4.

3.3.1 Dropping probability approximation

Note that for the limiting case of no mobility, the FT and TB models are identical, and $d^{\mathrm{ft}} = d^{\mathrm{tb}} = 0$. In the case of mobility, we shall compare in Sect. 4 the value of d^{ft} obtained by simulations of the FT model versus d^{tb}. This comparison shows that

$$d^{\mathrm{ft}} \approx d^{\mathrm{tb}} \tag{6}$$

when $d^{\mathrm{tb}} < 0.02$. The limit of validity of the above approximation is enough for practical needs since in real-life networks the dropping probability should be maintained less than 10^{-3}. In the particular case when Eq. 5 holds true, the approximation (6) gives

$$d^{\mathrm{ft}} \approx \frac{2\upsilon}{\pi R\mu} b^{\mathrm{tb}}.$$

If moreover the arrival intensity is proportional to the mobility stationary distribution, then we deduce from Remark 1 that $b^{\mathrm{tb}} = b_0$ (the blocking probability evaluated for $\upsilon = 0$). Thus

$$d^{\mathrm{ft}} \approx \frac{2\upsilon}{\pi R\mu} b_0 \tag{7}$$

where υ is the user's average speed, R is the cell radius and b_0 is the blocking probability of the no-mobility case.

3.3.2 Blocking probability bound

The following reasoning is heuristic. As the user speed increases, we deduce from the approximation (7) that the dropping probability increases and thus more calls are interrupted leaving more resources free for new arriving calls. Thus the (access) blocking probability would decrease. In particular it is bounded above by b_0, that is

$$b^{\mathrm{ft}} \leq b_0 \tag{8}$$

Remark 2 We may use b_0 as an approximation of the blocking probability in the FT model, that is

$$b^{\mathrm{ft}} \simeq b_0$$

This approximation leads to a *safe* dimensioning (i.e., the blocking really perceived by the users would be smaller than the target value used in dimensioning).

4 Streaming in OFDMA

In this section we will validate the approximations proposed in the previous section by simulations of an OFDMA cellular network.

4.1 Feasible configurations of users

It is natural to identify the feasible configurations of users in the network studying the feasibility of the resource (power and bandwidth) allocation problem.

In this approach, a given configuration of users with predefined bit-rates is *feasible* if there exists a resource allocation which respects the information theory constraint as well as the maximal power and total bandwidth constraints (see [7] for more details). However, solving the resource allocation problem for a large network is a very complicated task.

It is shown in [7] that a sufficient condition for the feasibility of resource allocation is that each base station u respects the following inequality for the users of its own cell

$$\int_u \varphi_u(x)\nu(dx) = \sum_{x\in\nu\cap u} \varphi_u(x) < 1 \tag{9}$$

where $\varphi_u(\cdot)$ is some nonnegative function of the user x location and bit-rate. The above approach suggests the following conservative choice for the set of feasible configurations $\mathbb{M}^{\mathrm{f}}$ of users

$$\mathbb{M}^{\mathrm{f}} = \nu \in \mathbb{M} : \int_u \varphi_u(x)\nu(dx) < 1, \forall \text{cell } u. \tag{10}$$

Remark 3 We shall assume AWGN channels between the BSs and users. In this case, the function $\varphi_u(\cdot)$ in (9) equals (cf [7])

$$\varphi_u(x) = \frac{r_x}{W \log_2\left(1 + 1/\hat{f}(x)\right)}$$

where W is the system bandwidth, r_x is the bit-rate of user x, and

$$\hat{f}(x) = \frac{1}{1-e}\left(\frac{NL_{u,x}}{\tilde{P}} + f(x)\right), \quad x \in u$$

where $\tilde{P}$ is the BS maximal power, e is the fraction of the this power used by common channels, N is the noise power, $L_{u,x}$ is the propagation loss and

$$f(x) = \sum_{v\neq u} \frac{L_{u,x}}{L_{v,x}}, \quad x \in u$$

is the interference factor or f-factor. (For efficient methods to calculate the f-factor cf [20].)

4.2 Discretization

In order to get a discrete model, we partition each cell into several rings around the base station. Then we take as representative value of the function $\varphi_u(\cdot)$ in each ring the average of $\varphi_u(x)$ where x is distributed according to $\rho(dx)$ within the considered ring. In this case the TB blocking probability can be evaluated via the *Kauffman-Roberts algorithm* [27].

In the most simple case, when u is not partitioned at all, we get a unique value, denoted $\bar{\varphi}_u$, which equals

$$\bar{\varphi}_u = \int_u \varphi_u(x)\rho(dx)/\rho(u).$$

Thus we get the following Erlang-type approximation of $\mathbb{M}^{\mathrm{f}}$

$$\mathbb{M}^{\mathrm{f}} = \{\nu \in \mathbb{M} : \nu(u) \leq 1/\bar{\varphi}_u, \forall \text{cell } u\} \tag{11}$$

In this case, the TB blocking probability may be calculated by using the Erlang's formula.

We will distinguish two variants of the mobility model:

- In the *intercell mobility model*, only the mobility between the cells is taken into account. This model is suitable when the feasibility condition depends only on the number of users in each cell as in (11).
- The *complete mobility model* comprises both mobility within each cell and between the different cells. This model is relevant when the feasibility condition depends on the specific positions of the users within each cell as in (10).

The explicit expressions for the mobility kernel for the two above variants of the *completely aimless mobility* are given in "Appendix" by Eqs. 18 and 19 respectively.

4.3 Model specification

4.3.1 Network architecture

We consider the radio part of the downlink in wireless cellular OFDMA networks. In order to obtain numerical values, we consider the most popular hexagonal model, where the base stations (BS) are placed on a regular hexagonal grid. Let R be the radius of the disc whose area is equal to that of the hexagonal cell and call R the *cell radius*. We take $R = 0.525$, 3 or 5 km. In order to avoid the boundary effects we consider the network that is "wrapped around"; i.e., deployed on a torus comprising $4 \times 4 = 16$ cells. Each cell is decomposed into three equally thick rings around the BS.

The system bandwidth equals $W = 5$ MHz. The ambient noise power equals $N = -103$ dBm. BS are equipped with omnidirectional antennas. The BS maximal power equals $\tilde{P} = 52$ dBm. The common channel power $\hat{P} = e\tilde{P}$ where $e = 0.12$.

We assume a propagation loss $L(r) = (Kr)^{\eta}$, with $\eta = 3.38$ and $K = 8667$ where r is the distance between the transmitter and the receiver.

Remark 4 Note that we don't account for *shadowing* in our numerical application. A first simple way to account for shadowing is to assume that it modifies the geometry of the cells, but that this geometry remains fixed at the time scale of the dynamics of call arrivals and mobility. In this case, some results of the present paper such as (6) are sufficiently general to hold true also in this case. Moreover, there is no obvious reason so that an approximation such as (7) is not accurate any more. Nevertheless, it is interesting in future work to check (7) in the case of shadowing.

4.3.2 Arrivals and call durations

We consider streaming traffic with the required bit rate 180 Kbps. We take a mean call duration $1/\mu = 2$min. We assume the (spatially) uniform arrival stream $\lambda(dx) = \lambda\, dx$ with λ varying such that the *traffic demand* $\lambda\pi R^2/\mu$ varies from 0 to 40 Erlangs per cell.

4.3.3 User mobility

We assume the *completely aimless* mobility model described in "Appendix". By Proposition A.3, the mobility stationary distribution σ is uniform. In view of the above arrival/call duration specification and of Proposition 3, the traffic intensity is $\rho(dx) = (\lambda/\mu)dx$. We will consider three values of the *mean user speed* $\upsilon = 0.1$, 1, 3*km per mean call duration*, which correspond, respectively, to $\upsilon = 3$, 30 and 90km/h for the considered mean call duration $1/\mu = 2$min.

We consider both the *intercell* and *complete* mobility models with the feasibility sets given by (11) and (10) respectively.

4.4 Numerical results

We first give the results for the intercell mobility model, then those for complete mobility.

4.4.1 Intercell mobility

Study of the TB model: Note first that the arrival intensity and the mobility stationary distribution are both spatially uniform. Then we deduce from Remark 5, that the blocking probability b^{tb} does not depend on the user speed υ. That is $b^{\text{tb}} = b_0$ (the blocking probability of the no-mobility case) which may be calculated by using the Erlang's formula. The results of our simulations confirm these theoretical results.

We also check that the simulations results for the mean number of motion blocking per call d^{tb} fit well with our analytical formula (5). We do not report the figures corresponding to this check to save space.

Study of the FT model: Figure 1 shows the dropping probability d^{ft} obtained by simulations of the FT model versus the mean number of motion blocking per call d^{tb} in the TB model. We see that there is a good fit between d^{ft} and d^{tb} as long as they remain less than 0.02. In real-life networks the dropping probability should not exceed 10^{-3}. Thus the proposed approximation (6) is suitable for practical needs.

4.4.2 Complete mobility

Study of the TB model: As for the case of intercell mobility, the blocking probability b^{tb} does not depend on the user speed υ; that is $b^{\text{tb}} = b_0$. But in the present case,

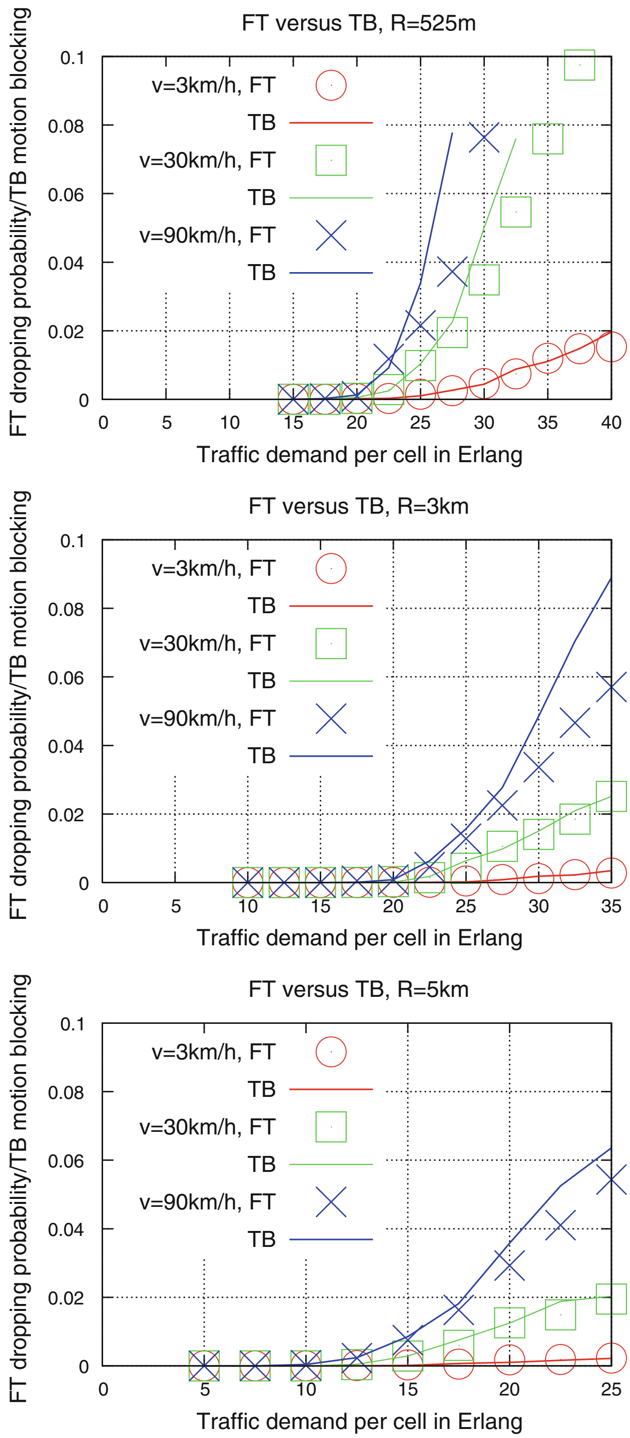

Fig. 1 Simulated dropping probability d^{ft} versus its approximation d^{tb} for intercell mobility

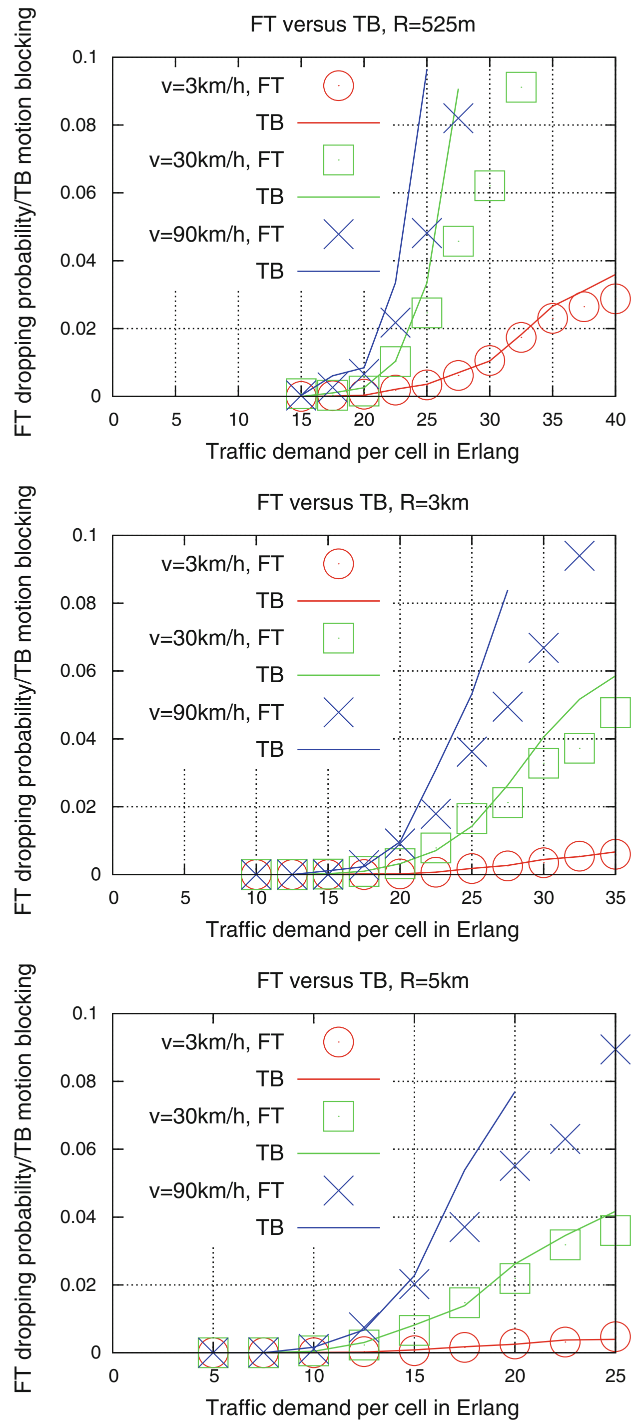

Fig. 2 Simulated dropping probability d^{ft} versus its approximation d^{tb} for complete mobility

the *Kauffman-Roberts algorithm* [27] (instead of Erlang's formula) permits to calculate b_0. The results of our simulations confirm these theoretical results.[8]

[8] Here also we don't report the numerical results not to increase the number of figures unnecessarily.

Study of the FT model: Figure 2 shows the dropping probability d^{ft} obtained by simulations of the FT model versus the mean number of motion blocking per call d^{tb} in the TB model. We see that approximation (6) is also suitable for the complete mobility model.

We observe that, for a given traffic demand, the dropping probability increases with the mean user speed. We observe moreover that the dropping probability for complete mobility is larger than that for intercell mobility. This is intuitively related to the fact that dropping may occur due to intracell mobility in the former model, while such dropping does not occur in the latter one.

Figure 3 shows the blocking probability b^{ft} obtained by simulations of the FT model. Observe that the blocking probability decreases when the mean user speed increases. This validates numerically the bound (8). (An analogous result is also obtained for intercell mobility.)

Remark 5 The results reported in [6] for CDMA are less detailed than the above ones for OFDMA. (In particular, intercell and complete mobility models are not clearly distinguished in [6]). Nevertheless, the results of the two studies are coherent, and thus the proposed approach seems to apply to a wide class of cellular networks.

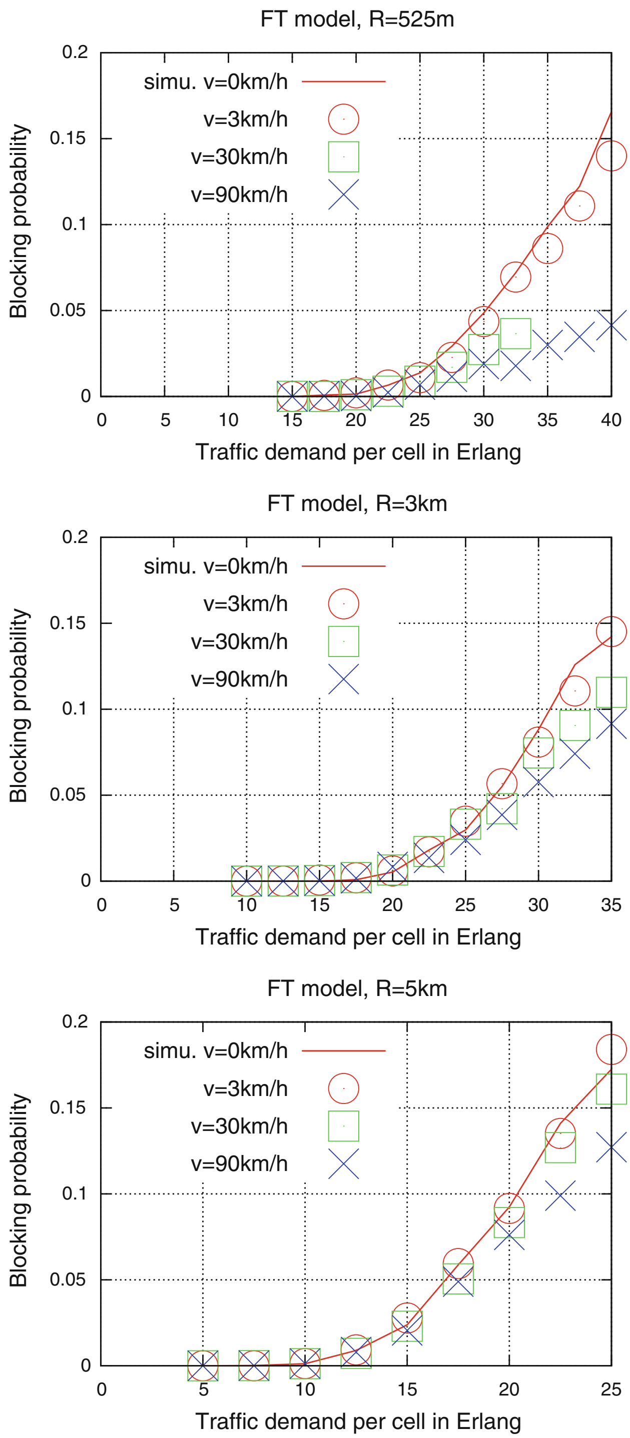

Fig. 3 Blocking probability b^{ft} for complete mobility

5 Conclusion

We investigate the impact of mobility of streaming users on their QoS in wireless cellular networks. To do so we study two models called *forced termination* (FT) and *transitions backtrack* (TB). When a user's motion leads to a violation of the capacity constraint, then his call is dropped in the FT model whereas the user backtracks to his previous location in the TB model. The study of the two models gives an insight on the qualitative and quantitative dependence of the QoS on the mean user speed.

In the TB model, we establish analytical expressions for the *(access) blocking probability* that under some quite natural assumptions does not depend on the mean user speed. In this model we also analyze the *mean number of blocked motions per call* (i.e., number of times the user backtracks).

For the FT model, in which, besides call (access) blocking, one observes call dropping, we show that the *dropping probability* is well approximated by the mean number of blocked motions per call evaluated for the corresponding TB model. This gives in some particular cases a simple expression of the dropping probability as function of the user's speed, the cell radius and the blocking probability (of the motionless case). A simple bound for the *(access) blocking probability* in the FT model is also given. We validate this approximation and this bound by simulating an OFDMA network.

These results are very useful for dimensioning wireless cellular networks; i.e., evaluating the minimal number of base stations assuring that the blocking and dropping probabilities are less than a given threshold (for some given traffic demand and mean user's speed).

Acknowledgments The author thanks Prof. Bartlomiej Blaszczyszyn at INRIA for his precious encouragements and help.

Appendix A: Completely aimless mobility

We present in this appendix a mobility model based on the following assumptions (see [30]):

- The speeds of the users are considered as *random vectors* in $\mathbb{R}^2$ and are assumed independent and identically distributed.
- The speed *direction* of a typical user is a random variable which is uniformly distributed in $[0, 2\pi)$.

Following the authors of [18] we call this model *completely aimless* mobility and assume that the user's *sojourn duration* in a given zone has an exponential distribution.[9] We now express its average.

Proposition A.1 *Consider the completely aimless mobility model and denote by υ the average user speed. Consider a given geographic zone of area A and perimeter L. The average number of users crossing the zone border per time-unit is given by*

$$\bar{n} = \frac{\upsilon\rho}{\pi} L \tag{12}$$

The average user's sojourn duration in the zone is given by

$$\tau = \frac{\pi A}{\upsilon L} \tag{13}$$

Proof (See [30].) We are interested in the users crossing an infinitesimal element dl of the border (for example from outside to inside) within an infinitesimal duration dt. Such users are located in a rectangle of sides dl and $V\cos\alpha dt$, as illustrated in Fig. 4, where: V is the user's speed magnitude and α is the angle formed by the user's speed vector and the perpendicular to dl. Integrating over V and α, we obtain the average number of users crossing an element dl of the border of the zone, from outside to inside, during dt

$$\int_{-\pi/2}^{\pi/2}\int_0^{+\infty} VF(dV)\cos\alpha\frac{d\alpha}{2\pi}\rho dldt = \frac{\upsilon\rho}{\pi}dldt$$

where ρ is the density of users per surface unit, F is the cumulative distribution function of the user's speed magnitude and $\upsilon = E[V]$. This proves (12). Let $\bar{M} = \rho A$ be the average number of users in the zone. By Little's formula [5], we have

$$\bar{M} = \bar{n}\tau$$

which together with (12) gives (13). □

Intracell mobility

The cell is modeled by a disc of radius R which is divided into J rings. Each ring denoted by some $j \in \mathcal{J} = \{1,\dots,J\}$

[9] This assumption is justified in [12].

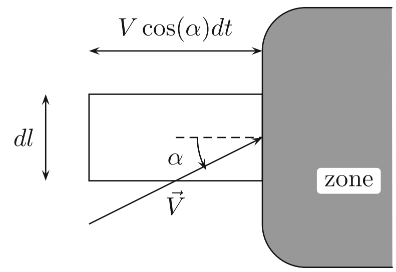

Fig. 4 Rectangle containing customers crossing an element *dl* of the border during *dt*

is delimited by discs with radii r_{j-1} and r_j where $r_0 = 0$ and $r_J = R$. Let $A_j = \pi\left(r_j^2 - r_{j-1}^2\right)$ be the surface of ring j. Of course J should be large enough to capture correctly the geometry of the problem.

Consider the case where mobility is within a given cell. Denote λ'_j the inverse of the average sojourn duration of users at ring j. Applying Eq. 13 gives

$$\begin{array}{ll} \lambda'_j = \frac{\upsilon L_j}{\pi A_j} = 2\upsilon\frac{r_j + r_{j-1}}{A_j}, & j = 1,\dots,J-1 \\ \lambda'_J = \frac{\upsilon L_J}{\pi A_J} = 2\upsilon\frac{r_{J-1}}{A_J} & \end{array}$$

A user finishing its sojourn at ring j is routed: - either to ring $j-1$ or to ring $j+1$ with respective probabilities $p'_{j,j-1} = r_{j-1}/\left(r_j + r_{j-1}\right)$ and $p'_{j,j+1} = r_j/\left(r_j + r_{j-1}\right)$, if $j = 2, \dots, J-1$;

- to ring 2 with probability 1, if $j = 1$;
- to ring $J-1$ with probability 1, if $j = J$.

Define the *mobility kernel* (λ_{jk}) on $\mathcal{J}$ by

$$\lambda_{jk} = \lambda'_j p'_{jk}, \quad j,k \in \mathcal{J}.$$

We deduce from the above results that

$$\begin{array}{ll} \lambda_{j,j-1} = 2\upsilon\frac{r_{j-1}}{A_j}, & j = 2,\dots,J \\ \lambda_{j,j+1} = 2\upsilon\frac{r_j}{A_j}, & j = 1,\dots,J-1. \end{array} \tag{14}$$

Proposition A.2 *The mobility kernel $\left(\lambda_{jk}, j,k \in \mathcal{J}\right)$ defined by (14) admits*

$$\sigma_j = \frac{A_j}{\pi R^2}, \quad j \in \mathcal{J} \tag{15}$$

as stationary distribution, i.e. $\left(\sigma_j, j \in \mathcal{J}\right)$ is solution of the following equations

$$\sigma_j \sum_k \lambda_{j,k} = \sum_k \sigma_k \lambda_{k,j}, \quad j \in \mathcal{J}. \tag{16}$$

Equation 16 may be written as follows

$$\begin{cases} \sigma_j\left(\lambda_{j,j-1} + \lambda_{j,j+1}\right) = \sigma_{j-1}\lambda_{j-1,j} + \sigma_{j+1}\lambda_{j+1,j} \\ \text{for } j = 2,\dots,J-1 \\ \sigma_1\lambda_{1,2} = \sigma_2\lambda_{2,1} \\ \sigma_J\lambda_{J,J-1} = \sigma_{J-1}\lambda_{J-1,J} \end{cases}$$

For the rates (14) we get

$$\begin{cases} \sigma_j \frac{r_j+r_{j-1}}{A_j} = \sigma_{j-1}\frac{r_{j-1}}{A_{j-1}} + \sigma_{j+1}\frac{r_j}{A_{j+1}} & \text{for } j = 2,\ldots,J-1 \\ \sigma_1\frac{r_1}{A_1} = \sigma_2\frac{r_1}{A_2} \\ \sigma_J\frac{r_J-1}{A_J} = \sigma_{J-1}\frac{r_{J-1}}{A_{J-1}} \end{cases}$$

which clearly admits σ given by (15) as solution.

Intracell mobility

Let λ'_u be the inverse of the average sojourn duration of users within cell u. Applying Eq. 13 we get

$$\lambda'_u = \frac{\upsilon L}{\pi A} = \frac{2\upsilon}{\pi R} \tag{17}$$

where the perimeter equals $L = 2\pi R$ and the area equals $A = \pi\ R^2$. If each base station has 6 neighbors as in the toric hexagonal model, then a user finishing its sojourn in cell u is routed to a neighboring cell v with probability

$$p'_{u,v} = \frac{1}{6}.$$

Define the mobility kernel $\lambda_{u,v}$ on the set of cells by

$$\begin{aligned} \lambda_{u,v} &= \lambda'_u p'_{u,v} \\ &= \frac{1}{6}\lambda'_u = \frac{\upsilon}{3\pi R} \end{aligned} \tag{18}$$

for each pair of neighboring cells u, v.

Complete mobility

Consider now a network of hexagonal cells such that each one has exactly 6 neighbors. Each cell is approximated by a disc and divided into J rings. The cells are indexed by $u \in \mathcal{U} = \{1,\ldots,U\}$, and the rings by $j \in \mathcal{J} = \{1,\ldots,J\}$. The ring j of the cell u is indexed by $uj \in \mathcal{U}\times\mathcal{J}$.

Denote λ'_{uj} the inverse of the average sojourn duration of users in the ring uj. Applying Eq. 13 gives

$$\lambda'_{uj} = \frac{\upsilon L_j}{\pi A_j} = 2\upsilon\frac{r_j + r_{j-1}}{A_j}, \quad u \in \mathcal{U}, j \in \mathcal{J}.$$

A user finishing its sojourn in ring uj is routed:

- to either ring $u(j-1)$ or ring $u(j+1)$ with respective probabilities $p'_{uj,u(j-1)} = r_{j-1}/(r_j + r_{j-1})$ and $p'_{uj,u(j+1)} = r_j/(r_j + r_{j-1})$, if $j = 2, \ldots, J-1$;
- to ring $u2$ with probability 1, if $j = 1$;
- to either ring $u(J-1)$ or ring vJ, where v is a neighbor of u, with respective probabilities $p'_{uJ,u(J-1)} = r_{J-1}/(r_J + r_{J-1})$ and $p'_{uJ,u(J+1)} = r_J/(r_J + r_{J-1})$, if $j = J$.

Define the *mobility kernel* $(\lambda_{uj,vk})$ on $\mathcal{U}\times\mathcal{J}$ by

$$\lambda_{uj,vk} = \lambda'_{uj}p'_{uj,vk}, \quad u,v \in \mathcal{U}, j,k \in \mathcal{J}.$$

We deduce from the above results that

$$\begin{cases} \lambda_{uj,u(j-1)} = 2\upsilon\frac{r_{j-1}}{A_j}, & j = 2,\ldots,J \\ \lambda_{uj,u(j+1)} = 2\upsilon\frac{r_j}{A_j}, & j = 1,\ldots,J-1 \\ \lambda_{uJ,vJ} = \frac{1}{3}\upsilon\frac{r_J}{A_J}, & v \text{is a neighbor of} u. \end{cases} \tag{19}$$

The result of Proposition A.2 may be easily extended to the complete mobility case as follows.

Proposition A.3 *The mobility kernel* $(\lambda_{uj,vk}; uj, u, v \in \mathcal{U}, j, k \in \mathcal{J})$ *given by* (19) *admits*

$$\sigma_{uj} = \sigma_j = \frac{A_j}{\pi R^2}, \quad u \in \mathcal{U}, j \in \mathcal{J}$$

as stationary distribution, i.e. $(\sigma_{uj}, u \in \mathcal{U}, j \in \mathcal{J})$ *is solution of the following equations*

$$\sigma_{uj}\sum_{v\in\mathcal{U},k\in\mathcal{J}}\lambda_{uj,vk} = \sum_{v\in\mathcal{U},k\in\mathcal{J}}\sigma_{vk}\lambda_{vk,uj}, \quad u \in \mathcal{U}, j \in \mathcal{J}.$$

Proof Besides the proof of Proposition A.2, it remains to show that

$$\sigma_J\left[\lambda_{uJ,u(J-1)} + \sum_v \lambda_{uJ,vJ}\right] = \sigma_{J-1}\lambda_{u(J-1),uJ} + \sigma_J\sum_v \lambda_{vJ,uJ}$$

which is equivalent to

$$\sigma_J\lambda_{uJ,u(J-1)} = \sigma_{J-1}\lambda_{u(J-1),uJ}$$

which holds true. □

Appendix B: Mathematical background

In this section we prove some mathematical results which are used in the paper. More on the mathematical background can be found in [19, Ch. 8].

Free process

Let $\mathbb{D}$ be a bounded subset of $\mathbb{R}^2$ designating the *set of users locations*. We introduce a "virtual" location $o \notin \mathbb{D}$ which can be seen as a location outside the space $\mathbb{D}$, from which users arrive to the system and which represents the destination of the users leaving the system. Denote $\overline{\mathbb{D}} = \mathbb{D}\cup\{o\}$.

Configurations $\{x_i\} \subset \mathbb{D}$ of users in the system are identified by corresponding counting measures $\nu = \sum_i \varepsilon_{x\_i}$; where the Dirac measure ε_x is defined by $\varepsilon_x(A) = 1$ if $x \in A$ and 0 otherwise. We denote by $\mathbb{M}$ the set of *all finite configurations of users* (i.e., finite counting measures) on $\mathbb{D}$.

The temporal evolution of the configuration of users in $\mathbb{D}$ is described by a continuous-time*jump Markov* process, which takes values in $\mathbb{M}$. This process evolves because of users arriving, moving or leaving the system, with only one such event being possible at a time. Assume given the *generator* q of such process (i.e., we assume given $q(v, \Gamma)$ for each state $v \in \mathbb{M}$ and each measurable $\Gamma \subset \mathbb{M}$, but we do not assume any particular expression for them) which we call *free process*.

It will be helpful to introduce the following *operator* T on the space $\mathbb{M}$: for $v \in \mathbb{M}, x \in v, y \in \mathbb{D}$: $T_{oy}v = v + \varepsilon_y$, $T_{xo}v = v - \varepsilon_x$, and $T_{xy}v = v - \varepsilon_x + \varepsilon_y$ corresponding respectively to arrivals, departures and mobility. It is customary to define also

$$T_{AB}v = \{T_{xy}v : x \in A, y \in B, x \neq y\}$$

for $A, B \subset \overline{\mathbb{D}}, v \in \mathbb{M}$.

Consider a fixed (measurable) subset $\mathbb{M}^{\mathrm{f}} \subset \mathbb{M}$. We call $\mathbb{M}^{\mathrm{f}}$ the set of *feasible* states. We assume that if $v' \in \mathbb{M}^{\mathrm{f}}$ then for any $v \subset v'$ one has $v \in \mathbb{M}^{\mathrm{f}}$ (*monotonicity property of* $\mathbb{M}^{\mathrm{f}}$).

Transitions backtrack process

We associate to the free process a *transitions backtrack* (TB) process defined by its generator q^{tb} being the truncation of q to $\mathbb{M}^{\mathrm{f}}$; that is

$$q^{\mathrm{tb}}(v, \Gamma) = \begin{cases} q(v, \Gamma \cap \mathbb{M}^{\mathrm{f}}) & \text{if } v \in \mathbb{M}^{\mathrm{f}}, \Gamma \subset \mathbb{M} \\ q(v, \Gamma) & \text{if } v \in \mathbb{M} \setminus \mathbb{M}^{\mathrm{f}}, \Gamma \subset \mathbb{M} \end{cases}$$

Suppose that q is ergodic and denote its stationary distribution by Π. In what follows we assume that q^{tb} is also ergodic and has a particular form of the stationary distribution $\Pi^{\mathrm{tb}}(\cdot) = \Pi(\cdot \cap \mathbb{M}^{\mathrm{f}})/\Pi(\mathbb{M}^{\mathrm{f}})$ being the *truncation* of Π to $\mathbb{M}^{\mathrm{f}}$. This truncation property does not always hold, and one simple sufficient condition for this to hold is when the original free process given by q is reversible (see [28, Proposition 3.14]).

Blocking

Consider the TB process $\{N_t\}$ with generator q^{tb}. In order to formalize the notion of the blocking probability and blocked displacements one models the time-epochs and departure-arrival locations of these blocked transitions by a double stochastic Poisson point process. More specifically, let

$$\Phi_0 = \sum_i \varepsilon_{(t_i, x_i, y_i)}$$

where t_i, x_i, y_i denote, respectively, the time-epochs, departure and arrival locations of blocked transitions of $\{N_t\}$. Given a realization $\{N_\cdot\}$ of the TB process, Φ_0 is a Poisson point process with intensity measure $\Lambda_{N_\cdot}$ on $(0, \infty) \times (\overline{\mathbb{D}})^2$, given by

$$\Lambda_{N_\cdot}(D \times A \times B) = \int_D q(N_t, T_{AB}N_t \setminus \mathbb{M}^{\mathrm{f}})dt.$$

Denote also by Φ_1 the point process on $(0, \infty) \times (\overline{\mathbb{D}})^2$ associated to (“ true”) transitions of N_t; i.e.,

$$\Phi_1(D \times A \times B) = \sum_{s > 0} 1(s \in D, N_s = T_{xy}N_{s-}, x \in A, y \in B).$$

Let $\Phi = \Phi_0 + \Phi_1$ be the superposition of $\Phi_i, i = 0, 1$. Finally define the blocking probability for the transitions $v \to T_{AB}(v)$ for some $A, B \in \overline{\mathbb{D}}$ and $v \in \mathbb{M}^{\mathrm{f}}$ (we will call them transitions from A to B for short) as the following limiting ratio of blocked transitions to all transitions

$$p^{\mathrm{tb}}_{AB} = \lim_{t \to \infty} \frac{\Phi_0((0, t] \times A \times B)}{\Phi((0, t] \times A \times B)} \tag{20}$$

The above limit exists by the following result.

Lemma B.1 *Suppose that* $\emptyset$ *is a positive recurrent state for* q^{tb} *(which is true in particular if* q^{tb} *is ergodic) with the limiting distribution* Π^{tb}. *If*

$$E_{\Pi^{\mathrm{tb}}}[q(N, \mathbb{M})] < \infty \tag{21}$$

then

$$\lim_{t \to \infty} \frac{1}{t} \Phi_0((0, t] \times A \times B) = E_{\Pi^{\mathrm{tb}}}[q(N, T_{AB}N \setminus \mathbb{M}^{\mathrm{f}})]$$

and

$$\lim_{t \to \infty} \frac{1}{t} \Phi_1((0, t] \times A \times B) = E_{\Pi^{\mathrm{tb}}}[q(N, T_{AB}N \cap \mathbb{M}^{\mathrm{f}})]$$

a.s. for any initial value $N_0 = v$ *of the TB process, for which the return time to* $\emptyset$ *is a.s. finite.*

Proof Consider a probability space on which the TB process $\{N_t\}_t$ and both point processes Φ_i $(i = 0, 1)$ are (time) stationary. Denote by $\mathbf{E}_{\Pi^{\mathrm{tb}}}$ the expectation corresponding to the stationary distribution of $\{N_t\}_{t \geq 0}$.

Condition (B.10) implies that the point process Φ_1 has finite intensity. Indeed,

$$\begin{aligned} E_{\Pi^{\mathrm{tb}}}[\Phi_1((0, 1] \times \overline{\mathbb{D}} \times \overline{\mathbb{D}})] &= E_{\Pi^{\mathrm{tb}}}[q^{\mathrm{tb}}(N_0)] \\ &\leq E_{\Pi^{\mathrm{tb}}}[q(N_0, \mathbb{M})] < \infty \end{aligned}$$

where the equality follows from the Lévy’s formula [23, p.232]. Similarly, the intensity of Φ_0 that is a doubly stochastic Poisson point process is finite

$$\begin{aligned} E_{\Pi^{\mathrm{tb}}}[\Phi_0((0, 1] \times \overline{\mathbb{D}} \times \overline{\mathbb{D}})] &= \int_0^1 E_{\Pi^{\mathrm{tb}}}[q(N_t, \mathbb{M} \setminus \mathbb{M}^{\mathrm{f}})]dt \\ &\leq E_{\Pi^{\mathrm{tb}}}[q(N_0, \mathbb{M})] < \infty. \end{aligned}$$

□

For given $A, B \subset \overline{\mathbb{D}}$ the processes $X^i_t = \Phi_i((0, t] \times A \times B)$ $(i = 1, 2)$ are cumulative with the imbedded renewal process being the epochs of successive visits of N_t at $\emptyset$. Thus

$$\lim_{t\to\infty}\frac{1}{t}\Phi_1((0,t]\times A\times B) = E_{\Pi^{\mathrm{tb}}}[\Phi_1((0,1]\times A\times B)]$$
$$= E_{\Pi^{\mathrm{tb}}}[q(N_0, T_{AB}N_0\cap\mathbb{M}^{\mathrm{f}})],$$

where the second equality follows from Lévy's formula. Similarly, by the fact that Φ_0 is a doubly stochastic Poisson point process

$$\lim_{t\to\infty}\frac{1}{t}\,\Phi_0((0,t]\times A\times B) = E_{\Pi^{\mathrm{tb}}}[\Phi_0([0,1]\times A\times B)]$$
$$= E_{\Pi^{\mathrm{tb}}}[\Lambda_{N_\cdot}((0,1]\times A\times B)]$$
$$= E_{\Pi^{\mathrm{tb}}}[q(N_0, T_{AB}N_0\setminus\mathbb{M}^{\mathrm{f}})].$$

This completes the proof.

The following result immediately follows from Lemma B.1.

Proposition B.4 *If the conditions of Lemma B.1 are satisfied, then*

$$p^{\mathrm{tb}}_{AB} = \frac{E^{\mathrm{tb}}_{\Pi}[q(N, T_{AB}N\setminus\mathbb{M}^{\mathrm{f}})]}{E_{\Pi^{\mathrm{tb}}}[q(N,T_{AB}N)]}.$$

Define the number of blocked displacements per user as follows

$$d^{\mathrm{tb}} = \lim_{t\to\infty}\frac{\Phi_0((0,t]\times\mathbb{D}\times\mathbb{D})}{\Phi((0,t]\times\mathbb{D}\times\{o\})}$$

The following result follows also from Lemma B.1.

Proposition B.5 *If the conditions of Lemma B.1 are satisfied then*

$$d^{\mathrm{tb}} = \frac{E^{\mathrm{tb}}_{\Pi}[q(N, T_{\mathbb{D}\mathbb{D}}N\setminus\mathbb{M}^{\mathrm{f}})]}{E_{\Pi^{\mathrm{tb}}}[q(N,T_{\mathbb{D}o}N)]}.$$

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Mohamed Kadhem Karray received his diploma in engineering from Ecole Polytechnique and Ecole Nationale Supérieure des Télécommunications (ENST) in 1991 and 1993, respectively. He prepared a PhD thesis at ENST under the guidance of Eric Moulines and Bartek Blaszczyszyn within 2004–2007. Since 1993 he works at France Telecom R&D (Orange Labs) in France. He co-authered together with François Baccelli and Bartek Blaszczyszyn publications in scientific journals and international conferences, and together with them he hold two patents on the load control in cellular networks. His research activities aim to evaluate the performance of communication networks. His principle tools are probability and stochastic processes, and more specifically information and queueing theories. In his recent research, he shows how to articulate the tools of these two theories to build global analytical performance evaluation methods for wireless cellular networks. The methods and tools he develops are used by Orange Operator for dimensioning its networks and for several practical studies such as the effect of a reduction of the transmitted power.