Electromagnetic Exposure and Quality of Service in the Downlink of Wireless Cellular Networks

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***Abstract*—We study in this paper both the *electromagnetic exposure* and the *quality of service* perceived by the users in the downlink of wireless cellular networks. We calculate the cumulative electromagnetic radiation received at each location from the interfering base stations in a *large* (eventually *infinite*) hexagonal cellular network. We establish a lower bound of the cell radius above which the *safety zone* has not to be increased due to this cumulative effect compared to the situation when one accounts only for the power received from the serving base station. On the other hand, fixing some quality of service target, we calculate the minimal emitted power required to serve a given traffic demand density with cells of a given radius. This permits to see whether the operator may *reduce* the power emitted currently in some parts of his network without degrading the quality of service. This is particularly interesting in the perspective of a potential reduction of the regulatory exposure threshold. Thus the present study shows when and of how much can be reduced the exposure of the people without sacrificing the users quality of the service.**

***Keywords*-Electromagnetic Exposure, Quality of Service, Downlink, Wireless Cellular Networks, Power.**

I. Introduction

We study in this paper both the *exposure* of the persons to the *electromagnetic waves* emitted by the base stations (BS) and the quality of service (QoS) perceived by the users in wireless cellular networks. We consider either Code-Division Multiple Access (CDMA) networks such as Universal Mobile Telecommunications System (UMTS) or Orthogonal Frequency-Division Multiple Access (OFDMA) networks such as 3GPP Long Term Evolution (LTE) with carrier frequency 2GHz.

The set of geographic locations for which the electromagnetic exposure is below the regulatory threshold is called *safety zone*. Assuming that the electromagnetic radiation received at a given location decreases with the distance to the serving base station, this zone corresponds to locations beyond some *safety distance*. This distance is already known when only the radiation received from the serving BS is taken into account. The impact of the radiation received from all the base stations in a *large* (eventually *infinite*) cellular network on the safety distance has not already been studied to our knowledge. Our *first objective* is to study this cumulative effect and to see *whether the safety distance has to be increased*.

Note that the power received at each location is roughly proportional to the powers emitted by the BS. Therefore we may attempt to reduce the received power by reducing the emitted powers. But in doing so we potentially degrade the QoS perceived by the users. Our *second objective* is to quantify the *minimal emitted power* required to assure a given QoS for a given traffic demand density with a network composed of cells with a given radius.

A key element to answer the above questions is the *relationship* between the *emitted power* and the users QoS, the traffic demand and the cell radius. Before looking for such relation, we have to define precisely the QoS perceived by the users. We consider *real-time* calls such as voice, real-time video, etc. Users arrive to the network at random times and locations, require some given bit-rate for some given duration, and depart from the network at the end of their calls. An important indicator of the QoS perceived by the users in such dynamic context is the *blocking probability* (defined as the proportion of the blocked calls to the total number of arriving calls in the long run of the system).

A. Related works

We discuss briefly the related work.

1) Electromagnetic Exposure: The International Commission on Non-Ionizing Radiation Protection (ICNIRP) has published a report [1] giving the maximum admissible exposure of the persons to the electromagnetic waves for different practical situations. More recently, ICNIRP published a report [2] on the exposure to high frequency electromagnetic waves and the resulting consequences on health.

Based on the ICNIRP recommendations, ANFR [3] gives the safety distances for people exposure for typical radio systems (in particular for GSM and UMTS) and for different configurations of antennas: mono or multi-band, on pylon or on house roof, etc. Joseph and Martens [4] compare the safety distance based on the electromagnetic field to that based on the specific absorption rate (SAR). Bornkessel et al. [5] give methods for the measurement and calculation of exposure around GSM and UMTS base stations. But these studies consider only the power received from the serving BS but not the cumulative power coming from the interfering BS that we aim to study in the present paper.

2) Performance: An extensive literature addresses performance of cellular networks [6], [7], [8] and [9]; but only few papers investigate the relation between the emitted power and the performance.

In such papers, the performance of wireless cellular networks is generally considered in terms of the *spectral efficiency* or the *outage probability* (i.e., the probability that the signal-to-interference ratio is less than some threshold) for a fixed number (or Poisson distribution) of users. With this definition of performance, it is already well known that *densification* increases the density of users served by a cellular network; see for example [10]. It is also known that *densification* permits to reduce the emitted power leading the network from interference-limited to noise-limited regime. This interesting result is pointed out by Liang et al. [11], but performance is also expressed there in terms of *outage*, which does not take into account the call arrivals and departures. Ramanath et al. [12] study also densification, but only for the uplink serving elastic bit-rate users.

Moreover, there is no papers, to our knowledge which study simultaneously the electromagnetic exposure and the performance.

B. Our contribution

We first calculate the cumulative power received at each location from the interfering BS in a *large* (eventually *infinite*) hexagonal cellular network. We investigate the impact of this cumulative power on the *safety zone* and particularly whether it has to be increased due to this cumulative effect.

On the other hand, by continuing work from [13] and [14], we show that the *blocking probability* may be viewed as a function of the emitted power, the cell radius and the traffic demand density (i.e., per surface unit). Fixing some blocking probability target, we represent the minimal emitted power as function of the cell radius for a given traffic demand density. This permits to see whether the operator may *reduce* the power emitted currently in some parts of his network without degrading the QoS. This is particularly interesting in the perspective of a potential reduction of the regulatory exposure threshold.

C. Paper organization

The remaining part of this paper is organized as follows. The cumulative power received at each location from the interfering base stations is studied in Section II. The relationship between the emitted power and the other key parameters (such as users QoS, traffic demand and cell radius) is studied in Section III. Numerical results are given Section IV.

II. Electromagnetic exposure

The electromagnetic radiation caused by some source in the *near-field* (i.e., in the source's neighborhood) have characteristics different from those in the *far-field* (i.e., sufficiently far away from the source). There are many conventions defining the transition between these two regions; we shall take the usual assumption [15, §.4.4] that the transition occurs at the so-called *Fraunhofer distance* defined by

$$r_0 = \frac{2D^2}{\lambda} \tag{1}$$

where λ is the wavelength and D is the largest dimension of the radiation source. More precisely, the near and far field regions correspond to distances from the radiation source below and beyond r_0 respectively.

Consider a network composed of a set of BS located on the plane $\mathbb{R}^2$. The electromagnetic radiation caused by the serving BS has already been studied (see for example [3] and [4], [5]). We aim in the present work to study the radiation caused by the *interfering* BS (i.e., all the BS other than the serving one).

We assume that each BS serves the locations which are closest to it than to any other BS (no shadowing). These locations constitute the *cell* associated to the BS. We assume that the distance between two adjacent BS is larger than two times the Fraunhofer distance; which is reasonable in view of the typical value of the Fraunhofer distance (17). Then the cell associated to a given BS is in the far-field region of the other BS.

The results in the following two subsections are already well known, but we recall them to make the paper self contained.

A. Exposure limit

The exposure of the persons to the electromagnetic waves in the *far-field* may be expressed in terms of the electric field strength. The power P_G *at the output* of a receiving antenna with gain G is related to the *electric field strength* E by the following classical relation [15, Chapter 2]

$$P_G = \frac{\lambda^2 G}{4\pi} \frac{E^2}{\eta_0}$$

where P_G is expressed in *Watt* (W), E is expressed in *Volt per meter* (V/m), λ is the wavelength in meters (m) and $\eta_0 = 377\text{ohm}$ (Ω).

The power p *at the input of the receiver* equals

$$p := \frac{P_G}{G} = \frac{\lambda^2}{4\pi} \frac{E^2}{\eta_0}$$

which we shall call *received power*. Letting p_{t} and E_{t} be the maximum admissible received power and electric field strength respectively, we deduce from the above equation that

$$p_{\text{t}} = \frac{\lambda^2}{4\pi} \frac{E_{\text{t}}^2}{\eta_0}. \tag{2}$$

B. Propagation loss

Assume that a BS transmits a power $\tilde{P}$, then the received power at a distance r from the considered BS equals $p(r) = \frac{\tilde{P}}{L(r)}$ where $L(r)$ is the propagation loss. The formulae for propagation loss due to distance are well known for the *far field* region. The worst situation for people exposure with regard to the distance loss is when the latter is *free space* propagation; i.e.,

$$L_0(r) = (K_0 r)^{\beta_0}, \quad r \geq r_0 \tag{3}$$

where $\beta_0 = 2$ and $K_0 = \frac{4\pi}{\lambda}$. If there is an obstacle between the BS and a given location then the free space model underestimates the propagation-loss. This is more likely to happen when the considered location is far from the base station. In this case, it is usual to take a propagation-loss having an expression similar to (3) with different values of the propagation parameters; that is

$$L(r) = (Kr)^{\beta}, \quad r \geq r_0 \tag{4}$$

where $\beta > 2$ and $K > 0$, as for example the Hata model [16].

C. Radiation caused by interfering base stations

We aim to calculate the power received from the interfering BS (i.e., all the BS other than the serving one). Let U be the set of BS composing the network. We denote $m \in u$ to say that location m is served by BS u. If each BS transmits a power $\tilde{P}$, then the power received at a location m from interfering BS equals

$$g(m) = \sum_{v \in U \setminus \{u\}} \frac{\tilde{P}}{L_{v,m}}, \quad m \in u.$$

where $L_{v,m}$ is the propagation-loss between BS v and location m.

We assume that the BS are placed on a regular *hexagonal* grid, which may be *infinite* on $\mathbb{R}^2$. Let Δ be the distance between two adjacent BS (inter-BS distance). Note that for a given BS u, the other BS are located on successive hexagons having u as center and having increasing radii (See Fig. 1). These hexagons are called *levels* and denoted $\mathcal{L}_1, \mathcal{L}_2, \ldots$. We may decompose $g(m)$ over these *levels* denoted $\mathcal{L}_1, \mathcal{L}_2, \ldots$ as follows

$$g(m) = \sum_{k \geq 1} g_k(m), \quad \text{where } g_k(m) := \sum_{v \in \mathcal{L}_k} \frac{\tilde{P}}{L_{v,m}}.$$

The following proposition expresses the *contribution* of the BS located on a given level in terms of the contribution of the BS located on the first one.

Proposition 1: For all $m \in u$, all $k \geq 1$,

$$g_k(m) = \sum_{l=0}^{k-1} \left(k^2 + l^2 - kl\right)^{-\beta/2} g_1\left(\frac{m e^{-i\frac{l\pi}{3k}}}{\sqrt{k^2 + l^2 - kl}}\right) \tag{5}$$

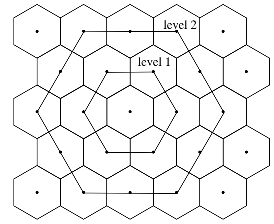

Figure 1. The first two levels $\mathcal{L}_1$ and $\mathcal{L}_2$.

where we identify $\mathbb{R}^2$ with the complex plane $\mathbb{C}$.

Proof: An inspection of Fig. 1 shows that $\mathcal{L}_k$ comprises $6k$ BS. We may decompose $\mathcal{L}_k$ into k groups of BS indexed by $l = 0, \ldots, k-1$; each group l is composed of 6 BS being at distance $\Delta\sqrt{k^2 + l^2 - kl}$ from the center. Consider the polar coordinates with respect to the central BS as origin. Then the angular coordinate of the first BS of each group l is $\frac{l\pi}{3k}$. The contribution of the l-th group may then be rearranged as follows $\left(k^2 + l^2 - kl\right)^{\beta/2} g_1\left(\frac{m}{\sqrt{k^2+l^2-kl}} e^{-i\frac{l\pi}{3k}}\right)$ which finishes the proof. ■

Proposition 2: For all $m \in u$,

$$\zeta(\beta - 1) \inf_{n \in u} g_1(n) \leq g(m) \leq \left(\frac{2}{\sqrt{3}}\right)^{\beta} \zeta(\beta - 1) \sup_{n \in u} g_1(n)$$

where ζ is the *Riemann zeta function* given by $\zeta(x) = \sum_{k=1}^{\infty} k^{-x}$.

Proof: Using the fact that $\frac{3}{4}k^2 \leq k^2 + l^2 - kl \leq k^2$ for all $k \geq 1$ and $0 \leq l \leq k - 1$ and the decomposition (5), we deduce that

$$k^{-\beta} \inf g_1 \leq g_k(m) \leq \left(\frac{2}{\sqrt{3}}\right)^{\beta} k^{-\beta} \sup g_1.$$

Adding the above inequality over all $k \geq 1$ finishes the proof. ■

Corollary 1: For all $m \in \mathbb{R}^2$,

$$g(m) \leq \left(\frac{2}{\sqrt{3}}\right)^{\beta} \frac{6\zeta(\beta - 1)\tilde{P}}{L(\Delta/2)} \tag{6}$$

where Δ is the inter-BS distance.

Proof: It is easy to see that for all $n \in u$, the distance between n and any BS $v \neq u$ is larger than $\Delta/2$. Thus $g_1(n) \leq \frac{6}{L(\Delta/2)}$. The upper bound in Proposition 2 finishes the proof. ■

The following corollary gives a condition assuring that the cumulative power received from the interfering BS $g(m)$ is less than some proportion, say δ, of the maximum admissible received power p_{t}.

Corollary 2: For a given constant $\delta > 0$, if

$$\Delta \geq \frac{4}{K\sqrt{3}} \left(\frac{6\zeta(\beta - 1)\tilde{P}}{\delta p_{\mathrm{t}}}\right)^{1/\beta} \tag{7}$$

then $g(m) \leq \delta p_{\mathrm{t}}$.

Proof: If the condition on Δ is satisfied then

$$L(\Delta/2) \geq \left(\frac{2}{\sqrt{3}}\right)^{\beta} \frac{6\zeta(\beta-1)\tilde{P}}{\delta p_{\mathrm{t}}}$$

which combined with (6) implies $g(m) \leq \delta p_{\mathrm{t}}$. ■

Taking δ sufficiently small (typically $\delta = 0.01$), the above Corollary gives the inter-BS distance above which the cumulative power received from the interfering BS may be neglected when evaluating the *safety zone*.

III. Quality of service

We now study the relationship between the emitted power and the other key parameters (such as users QoS, traffic demand and cell radius).

A. Model description

We will consider a wireless network composed of several BS serving some users. The propagation loss depends only on the distance between the transmitter and the receiver (no shadowing). We assume that each BS transmits a given *power* and that it serves users in some exclusive geometric cell associated to it, which *does not evolve in time*. In OFDMA networks we assume that each BS transmits a constant power spectral density.

Users perform *single user detection*; the interference is considered as noise. The bit-rate r of a given user is related to its bandwidth w and its SINR (signal to interference and noise power ratio) by

$$r \leq bw \log_2\left(1 + \frac{1}{a}\mathrm{SINR}\right) \tag{8}$$

where the constants a and b permit to account for the loss in practical systems compared to the ideal AWGN case (for which the above formula applies with $a = b = 1$). A value $b < 1$ may account for the loss of bandwidth due to signalling [17] and we may take $a \neq 1$ to account for the effect of fading on capacity as shown in [18].

Streaming calls arrive to the network and require to be served at a given bit-rate for some duration. The real-time traffic demand (expressed in Erlang) is defined as the ratio between the mean call duration and the mean inter-arrival time. If the network isn't able to satisfy the new arriving user together with the existing ones, then the new call is blocked. We assume that the *users don't move* during their service. We shall consider the *blocking probability* as a measure of the QoS perceived by the real-time users.

B. QoS evaluation method

We will show that a suitable admission condition in the considered networks has the following form:

$$\sum_{m\in u} \varphi(m) \leq 1 \tag{9}$$

for each BS u; where the sum is over the users m served by BS u, and $\varphi(m)$ is some function of the user location and bit-rate.

In order to express the function $\varphi(m)$ in a compact way, we introduce the following notation. Let U be the set of BS. We denote by $L_{u,m}$ the propagation-loss between BS u and user m (not including the fading) and by $\tilde{P}_u$ the total power emitted by BS u. We define the *interference factor* by

$$f(m) = \sum_{v\in U\setminus\{u\}} \frac{L_{u,m}}{L_{v,m}} \frac{\tilde{P}_v}{\tilde{P}_u}, \quad m \in u.$$

We introduce also the following slightly modified version of the interference factor

$$\hat{f}(m) = \frac{1}{1-\epsilon}\left(\frac{NL_{u,m}}{\tilde{P}_u} + \alpha + f(m)\right), \quad m \in u \tag{10}$$

where N is the noise power, α is the orthogonality factor which affects the intra-cell interference.and $\epsilon \in [0,1]$ is a fixed fraction of the total power used by the common channels (not dedicated to a specific user). Note that $\alpha = 0$ for OFDMA, whereas $\alpha \in (0,1)$ for CDMA.

1) CDMA: In a CDMA system, each user is allocated all the bandwidth W. The SINR of user $m \in u$ is equal to

$$\mathrm{SINR}_m = \frac{P_{u,m}/L_{u,m}}{N + \alpha\left(\tilde{P}_u - P_{u,m}\right)/L_{u,m} + \sum_{v\neq u} \tilde{P}_v/L_{v,m}}$$

where $P_{u,m}$ is the power allocated by BS u to user m. Combining the above equation with (8) we deduce that

$$P_{u,m} \geq (1-\epsilon)\tilde{P}_u \hat{f}(m) \frac{\xi_m}{1+\alpha\xi_m}, \quad m \in u \tag{11}$$

where $\hat{f}(m)$ is given by (10) and

$$\xi_m := a\left(2^{\frac{r_m}{bW}} - 1\right)$$

A power allocation $(P_{u,m})_{m\in u}$ satisfying (11) and $\sum_{m\in u} P_{u,m} \leq (1-\epsilon)\tilde{P}_u$ exists iff (9) holds true with

$$\varphi(m) = \hat{f}(m)\frac{\xi_m}{1+\alpha\xi_m} \tag{12}$$

2) OFDMA: Consider now an OFDMA network. Since each BS transmits a constant power spectral density, it allocates to each user m a power $P_{u,m}$ proportional to its bandwidth w_m; that is

$$P_{u,m} = \frac{w_m}{W}(1-\epsilon)\tilde{P}_u, \quad m \in u, u \in U \tag{13}$$

On the other hand, the SINR of user $m \in u$ is equal to

$$\mathrm{SINR}_m = \frac{P_{u,m}/L_{u,m}}{\frac{w_m}{W}N + \frac{w_m}{W}\sum_{v\neq u}\tilde{P}_v/L_{v,m}} = \frac{1}{\hat{f}(m)}$$

where for the second equality we use (13) and (10). Combining the above equation with (8) we deduce that

$$r_m \leq bw_m \log_2\left(1 + 1/\left(a\hat{f}(m)\right)\right), \quad m \in u$$

A bandwidth allocation $(w_m)_{m\in u}$ satisfying the above constraint and $\sum_{m\in u} w_m \leq W$ exists iff (9) holds true with

$$\varphi(m) = \frac{r_m}{bW\log_2\left[1+1/\left(a\hat{f}(m)\right)\right]} \qquad (14)$$

3) Blocking probability evaluation: We consider the hexagonal model, where the BS are placed on a regular hexagonal grid. The network is decomposed into J bins of surfaces $s_j, j = 1\ldots, J$ which are small enough to capture correctly the geometry of the problem. We consider only real-time calls whose inter-arrival times to bin j are independent and identically distributed (i.i.d.) exponential random variables with rate λ_j (mean $1/\lambda_j$). Each call requires to be served by the network at a given bit-rate during some service time. The durations of the different calls are assumed to be i.i.d. exponentially distributed with mean $1/\mu$. The vector ρ defined by $\rho_j = \frac{\lambda_j}{\mu s_j}, j = 1,\ldots, J$ is called *traffic demand density* (expressed in Erlangs per surface unit).

In the case of the admission condition (9), using the above cell discretization we may evaluate the blocking probability by using the Kaufman-Roberts algorithm [19], [20].

C. Power versus radius

The blocking probability $\mathbf{b}$ is a function of the cell radius R, the traffic demand density ρ and the emitted power $\tilde{P}$, which we write $\mathbf{b}\left(R,\rho,\tilde{P}\right)$. Then for a given blocking probability threshold $\mathbf{b}_0$, solving

$$\mathbf{b}\left(R,\rho,\tilde{P}\right) = \mathbf{b}_0 \qquad (15)$$

we get an implicit expression of the emitted power as function of the cell radius and the traffic demand density. We focus now our attention on this function. We present two observations which are useful to interpret the numerical results below.

1) Power lower bound: If there is a unique user m in cell u, then the admission condition (9) becomes

$$\varphi(m) \leq 1, \quad m \in u$$

which may be seen as a *coverage* condition. It is then natural to require that the above condition always holds true. Using the expressions (12) and (14) of $\varphi(m)$, the above condition writes $\hat{f}(m) \leq 1/\hat{\xi}_m$; or equivalently,

$$\tilde{P} \geq \frac{NL_{u,m}}{(1-\epsilon)/\hat{\xi}_m - \alpha - f(m)}. \qquad (16)$$

Thus the emitted power is lower bounded by the right-hand side of the above inequality. Note that this lower bound depends on the cell radius but not on the traffic demand.

2) Cell radius upper bound: Consider the theoretical case when the emitted power $\tilde{P} = \infty$, which is usually called the *pole* point (the capacity in this situation is called *pole capacity*). Then (15) becomes $\mathbf{b}(R,\rho,\infty) = \mathbf{b}_0$. Solving this equation, we may view the cell radius as an implicit function of the traffic demand density. We denote this function by $R_\infty(\rho)$.

IV. Numerical results

We present here the numerical results.

A. Electromagnetic Exposure

The largest dimension of antenna is typically $D \simeq 8\lambda$, then we deduce from (1) that the Fraunhofer distance $r_0 \simeq 128\lambda$; that is

$$r_0 \simeq 19.2\text{m} \qquad (17)$$

for a carrier frequency around 2GHz.

ICNIRP [1, Table 7] recommends $E_t = 61$V/m for general public exposure (for the considered carrier frequency 2GHz). Using (2), we deduce the maximum admissible received power $p_t = 0.022$W $(= 13.5$dBm$)$.

The Hata model [16] gives a propagation loss in the form (4) where $\beta = 3.38$ and $K = 8.667\text{m}^{-1}$ for an *urban* environment with BS height 50m and user height 1.5m. For *suburban* and *rural* environments with BS height 100m and user height 1.5m, we get $\beta = 3.18$ and $K = 1.612, 1.123\text{m}^{-1}$ respectively. Note that the Hata model has originally been built for the range of distances $r \in [1, 20]$km.

In order to study a large range of propagation parameters, we take $\beta \in [2, 5]$ and $K = 10, 2, 1\text{m}^{-1}$, which correspond typically to urban, suburban and rural environments respectively. We assume that the BS are equipped with antennas having a gain 12dBi and transmit a power 43dBm; thus $\tilde{P} = 43 + 12 = 55$dBm $(= 316$W$)$ when we account for antenna gain.

We aim now to study numerically the inter-BS distance (7) for which the cumulative power g received from the interfering BS doesn't exceed $0.01p_t$. Fig. 2 shows the cell radius (defined as half of the inter-BS distance; i.e., $\Delta/2$) as function of the propagation exponent β for $K = 10, 2, 1\text{m}^{-1}$ such that $g \leq 0.01p_t$. Observe that the cell radius decreases with β. Moreover for $\beta \geq 3$ and $K = 10, 2, 1\text{m}^{-1}$ we get cell radii $28, 140, 280$m respectively above which the cumulative power received from the interfering BS may be neglected when evaluating the *safety zone*.

B. Quality of service

We give now the numerical results showing the relationship between the emitted power and the other key parameters (such as users QoS, traffic demand and cell radius).

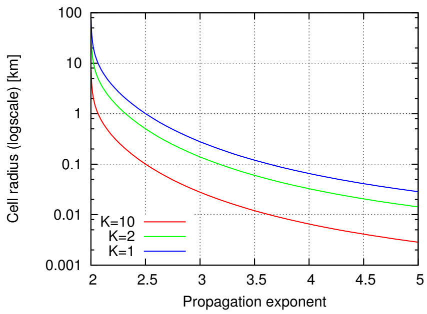

Figure 2. Cell radius such that $g \leq 0.01 p_{\mathrm{t}}$.

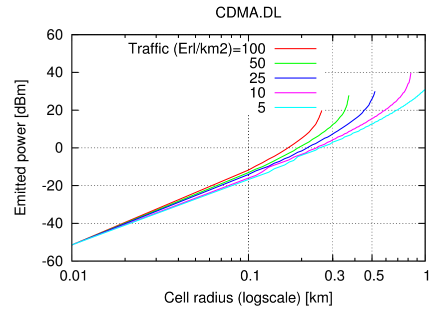

Figure 3. Emitted power as function of cell radius for CDMA.

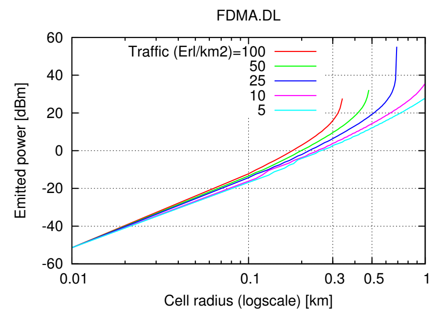

Figure 4. Emitted power as function of cell radius for OFDMA.

1) Model specification: We assume that the propagation loss is given by the Hata model for an *urban* environment. The system bandwidth equals $W = 5$MHz. The common channel power $\hat{P}$ is the fraction $\epsilon = 0.12$ of $\tilde{P}$ and the ambient noise power $N = -103$dBm. We consider real-time calls which may be either voice at 12.2Kbits/s or data at 64Kbits/s. In order to account for fading, we take an SNR reduction factor in the link performance formula (8) $a = 10$ and $b = 1$. (This leads to an SNR target of -18dB for voice and -11dB for data, which are typical in UMTS.) The traffic demand density ρ (in Erlang per surface unit) is composed of 90% of voice calls and 10% of data calls. In other words the traffic demand density equals 0.9ρ for voice and 0.1ρ for data.

2) Results: Fig. 3 and 4 show the emitted power $\tilde{P}$ (including antenna gains and losses) as function of the cell radius for different values of the traffic demand density for CDMA and OFDMA networks respectively. These curves permit to see whether the operator may *reduce* the power emitted currently in some parts of his network without degrading the QoS.

Observe that each of these curves has a vertical asymptote located at $R_{\infty}(\rho)$ defined in Section III-C2 and corresponding to the *pole point*. This means that for a given traffic demand density ρ, the cell radius should not exceed $R_{\infty}(\rho)$ otherwise the emitted power would be infinite.

Note also that curves for the different values of the traffic demand density are close to each other and become linear when the cell radius becomes small. Indeed this corresponds to a power equal to the right-hand side of (16); i.e., to the *coverage condition*. Thus densification brings the system from an interference limited regime (where the traffic density plays an important role) to a *noise limited regime* (where the traffic density does not intervene any more). This observation already made in [11] where performance is expressed in terms of the outage probability, is confirmed here where QoS is in terms of the blocking probability.

V. Conclusion

We calculate the cumulative power received at each location from all the interfering base stations in a *large* (eventually *infinite*) hexagonal cellular network. We establish a lower bound of the cell radius above which the *safety zone* has not to be increased due to this cumulative effect compared to the situation when one accounts only for the power received from the serving base station. For a propagation exponent $\beta \geq 3$, the numerical calculus gives a cell radius $28, 140, 280$m respectively for urban, suburban and rural environments respectively.

The *blocking probability* may be viewed as a function of the emitted power, the cell radius and the traffic demand density. Fixing some blocking probability target, we calculate the minimal emitted power required to serve a given traffic demand density with cells of a given cell radius. Then we represent the minimal emitted power as function of the cell radius for a given traffic demand density. This permits to see whether the operator may *reduce* the power emitted currently in some parts of his network without degrading the QoS. This is particularly interesting in the perspective of a potential reduction of the regulatory exposure threshold.

Note finally that the above quantification of the power as function of cell radius concerns a hexagonal network serving users who don't move during their calls and that the shadowing is not yet taken into account.

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