Electromagnetic Exposure and Quality of Service in the Downlink of Wireless Cellular Networks

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Paper [2]

Outline

Introduction

- Study both
  - *exposure* of the persons to *electromagnetic waves* emitted by the base stations (BS)
  - quality of service (QoS) perceived by the users
- In wireless cellular networks
  - Either CDMA networks such as UMTS
  - or OFDMA networks such as LTE
- *Safety distance* :
  - already known when radiation from serving BS is taken into account
  - impact of interfering BS in a *large* (eventually *infinite*) network has not already been studied
  - *First objective* : study this cumulative effect and see *whether the safety distance has to be increased*
- *Second objective* : quantify the *minimal emitted power* required to assure a given QoS for a given traffic demand density and cell radius

Electromagnetic exposure

- Transition between near and far fields occurs at *Fraunhofer distance*

$$r_0 = \frac{2D^2}{\lambda}$$

  where λ wavelength, D antenna largest dimension
- Typically $D \simeq 8\lambda$, then for a carrier frequency 2GHz

$$r_0 \simeq 19.2\text{m}$$

- We aim to study radiation caused by the *interfering* BS (i.e. all the BS other than the serving one)
- Assume that inter-BS distance larger than $2r_0$, then the cell of a given BS is in the far-field of other BS

Exposure limit

- Exposure in the *far-field* expressed in terms of *electric field strength* E
- Power P_G *at the output* of a receiving antenna with gain G

$$P_G = \frac{\lambda^2 G}{4\pi} \frac{E^2}{\eta_0}$$

where P_G in W, E in V/m, λ wavelength in m and $\eta_0 = 377\Omega$

- Power p *at the input of the receiver*

$$p := \frac{P_G}{G} = \frac{\lambda^2}{4\pi} \frac{E^2}{\eta_0}$$

called *received power*.

- Let p_{t} and E_{t} be the maximum admissible received power and electric field strength respectively

$$p_{\mathrm{t}} = \frac{\lambda^2}{4\pi} \frac{E_{\mathrm{t}}^2}{\eta_0}$$

Propagation loss

- If a BS transmits a power $\tilde{P}$, then received power at a distance r

$$p(r) = \frac{\tilde{P}}{L(r)}$$

where $L(r)$ is the propagation loss

- Formulae for propagation loss due to distance are well known for the *far field* region. Worst situation for people exposure is *free space*

$$L_0(r) = (K_0 r)^{\beta_0}, \quad r \geq r_0$$

where $\beta_0 = 2$ and $K_0 = \frac{4\pi}{\lambda}$

- If obstacles, then the free space model under-estimates the propagation-loss. In this case, it is usual to take

$$L(r) = (Kr)^{\beta}, \quad r \geq r_0$$

where $\beta > 2$ and $K > 0$, as for example the Hata model [1]

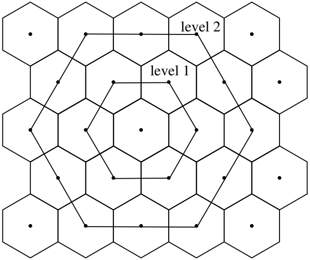
Radiation caused by *interfering* BS

- For m served by u, power received from interfering BS

$$g(m) = \sum_{v \neq u} \tilde{P}/L_{v,m}, \quad m \in u$$

  where $L_{v,m}$ loss between v and m, $m \in u$ means m served by u
- BS on *infinite hexagonal* grid. For a given BS u, the other BS are located on hexagons $\mathcal{L}_1, \mathcal{L}_2, \ldots$

$$g(m) = \sum_{k \geq 1} g_k(m), \quad \text{where } g_k(m) := \sum_{v \in \mathcal{L}_k} \tilde{P}/L_{v,m}$$



Radiation caused by *interfering* BS

- *Contribution* of BS in $\mathcal{L}_k$ in terms of $\mathcal{L}_1$

$$g_k(m) = \sum_{l=0}^{k-1} \left(k^2 + l^2 - kl\right)^{-\beta/2} g_1 \left(\frac{m e^{-i\frac{l\pi}{3k}}}{\sqrt{k^2 + l^2 - kl}} \right)$$

- Bounds

$$\zeta\left(\beta - 1\right) \inf_{n \in u} g_1\left(n\right) \le g(m) \le \left(\frac{2}{\sqrt{3}}\right)^{\beta} \zeta\left(\beta - 1\right) \sup_{n \in u} g_1\left(n\right)$$

where $\zeta\left(x\right) = \sum_{k=1}^{\infty} k^{-x}$ is the *Riemann zeta function*

- Let Δ be the inter-BS distance, then

$$g(m) \le \left(\frac{2}{\sqrt{3}}\right)^{\beta} \frac{6\zeta\left(\beta - 1\right)\tilde{P}}{L\left(\Delta/2\right)}$$

Radiation caused by *interfering* BS

- We deduce a condition assuring that $g(m)$ is less than some proportion, say δ, of the maximum admissible received power p_{t}
- For a given constant $\delta > 0$, if

$$\Delta \geq \frac{4}{K\sqrt{3}} \left(\frac{6\zeta(\beta-1)\tilde{P}}{\delta p_{\mathrm{t}}} \right)^{1/\beta}$$

  then $g(m) \leq \delta p_{\mathrm{t}}$
- If δ sufficiently small (typically $\delta = 0.01$), the above inequality gives the inter-BS distance above which the effect of interfering BS may be neglected when evaluating the *safety zone*

Model specification

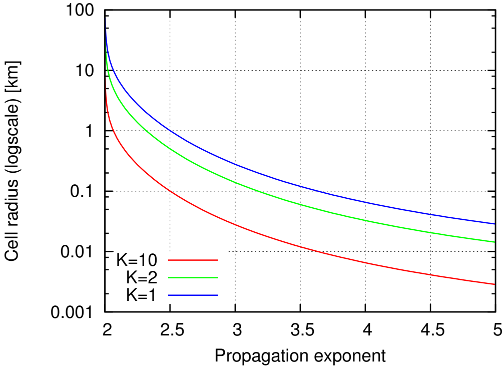
- ICNIRP recommends $E_{\rm t} = 61$V/m, thus the maximum admissible received power

$$p_{\rm t} = 0.022\text{W} \quad (= 13.5\text{dBm})$$

- Propagation parameters $\beta \in [2, 5]$ and $K = 10, 2, 1\text{m}^{-1}$ corresponding to urban, suburban and rural environments respectively
- Antennas gain 12dBi ; BS transmit a power 43dBm ; thus $\tilde{P} = 43 + 12 = 55$dBm $(= 316\text{W})$ when we account for antenna gain
- We plot cell radius (defined as $\Delta/2$) as function of β such that $g \leq 0.01 p_{\rm t}$

Cell radius such that $g \leq 0.01 p_{\mathrm{t}}$

- Cell radius decreases with β
- For $\beta \geq 3$ and $K = 10, 2, 1\mathrm{m}^{-1}$ we get cell radii $28, 140, 280\mathrm{m}$ respectively



Model description

- Propagation loss depends only on the distance (no shadowing)
- Each BS transmits a given *power* and a constant power spectral density
- Cells *don't evolve in time*
- Users perform *single user detection* ; the interference is considered as noise
- Bit-rate r related to bandwidth w and SINR

$$r \leq bw \log_2 \left(1 + \frac{1}{a} \text{SINR} \right) \quad (1)$$

- Real-time calls require to be served at a given bit-rate for some duration
- *Blocking probability* as a measure of the QoS

Admission condition

- We will show that a suitable admission condition has the multi-Erlang form

$$\sum_{m \in u} \varphi(m) \leq 1 \tag{2}$$

for each BS u; where $\varphi(m)$ is some function

- In order to express $\varphi(m)$, we introduce the *interference factor*

$$f(m) = \sum_{v \neq u} \frac{L_{u,m}}{L_{v,m}}, \quad m \in u.$$

and the modified version of the interference factor

$$\hat{f}(m) = \frac{1}{1-\epsilon} \left(\frac{N L_{u,m}}{\tilde{P}} + \alpha + f(m) \right), \quad m \in u$$

N noise power, α orthogonality factor, ϵ fraction of power used by common channels

CDMA

- SINR of user $m \in u$

$$\mathrm{SINR}_m = \frac{P_{u,m}/L_{u,m}}{N + \alpha\left(\tilde{P}_u - P_{u,m}\right)/L_{u,m} + \sum_{v \neq u} \tilde{P}_v / L_{v,m}}$$

  $P_{u,m}$ power allocated to user m.
- Combining the above equation with link constraint (1)

$$P_{u,m} \geq (1-\epsilon)\,\tilde{P}_u \hat{f}(m)\frac{\xi_m}{1+\alpha\xi_m}, \quad m \in u$$

  where

$$\xi_m := a\left(2^{\frac{r_m}{bW}} - 1\right)$$

- Such power allocation exists iff (2) holds true with

$$\varphi(m) = \hat{f}(m)\frac{\xi_m}{1+\alpha\xi_m}$$

OFDMA

- Each BS transmits a constant power spectral density

$$P_{u,m} = \frac{w_m}{W}(1-\epsilon)\tilde{P}_u, \quad m \in u, u \in U$$

- SINR of user $m \in u$

$$\mathrm{SINR}_m = \frac{P_{u,m}/L_{u,m}}{\frac{w_m}{W}N + \frac{w_m}{W}\sum_{v \neq u}\tilde{P}_v/L_{v,m}} = \frac{1}{\hat{f}(m)}$$

- Combining the above equation with link constraint (1)

$$r_m \leq b w_m \log_2\left(1 + 1/\left(a\hat{f}(m)\right)\right), \quad m \in u$$

- Such bandwidth allocation $(w_m)_{m\in u}$ exists iff (2) holds true with

$$\varphi(m) = \frac{r_m}{bW\log_2\left[1 + 1/\left(a\hat{f}(m)\right)\right]}$$

Blocking probability evaluation

- BS on a regular hexagonal grid. Network is decomposed into J bins of surfaces $s_j, j = 1 \ldots, J$
- Real-time calls whose inter-arrival times to bin j are i.i.d. exponential r.v. with rate λ_j
- Durations of different calls are i.i.d. exponentially distributed with mean $1/\mu$
- ρ defined by

$$\rho_j = \frac{\lambda_j}{\mu s_j}, \quad j = 1, \ldots, J$$

  is the *traffic demand density*
- We evaluate the blocking probability by using the Kaufman-Roberts algorithm

Power versus radius

- blocking probability $\mathbf{b}\left(R,\rho,\tilde{P}\right)$ function of cell radius R, traffic demand density ρ and emitted power $\tilde{P}$
- For a given blocking probability threshold $\mathbf{b}_0$, solving

$$\mathbf{b}\left(R,\rho,\tilde{P}\right)=\mathbf{b}_0$$

  we get an implicit expression of $\tilde{P}$ as function of R and ρ
- We present two observations which are useful to interpret the numerical results below.

Power lower bound

- If a unique user m in cell u, then (2) implies

$$\varphi(m) \leq 1, \quad m \in u$$

which may be seen as a *coverage* condition

- It is then natural to require that the above condition always holds true
- Using the expressions of $\varphi(m)$, the above condition writes

$$\tilde{P} \geq \frac{N L_{u,m}}{(1-\epsilon)/\hat{\xi}_m - \alpha - f(m)}.$$

Thus the emitted power is lower bounded

- This lower bound depends on the cell radius but not on the traffic demand

Cell radius upper bound

- Consider the theoretical case

$$\tilde{P} = \infty$$

  called usually the *pole* point
- Then solving

$$\mathbf{b}\left(R, \rho, \infty\right) = \mathbf{b}_0$$

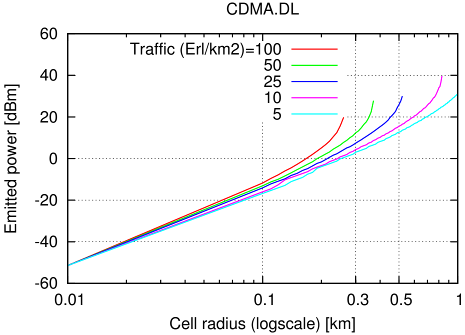
  we may view R as an implicit function of ρ
- Denote this function by $R_{\infty}\left(\rho\right)$

Model specification

- System bandwidth $W = 5$MHz ; common channel power is a fraction $\epsilon = 0.12$ of $\tilde{P}$; noise power $N = -103$dBm
- Real-time calls which may be either voice at 12.2Kbits/s or data at 64Kbits/s
- Account for fading in link performance formula (1) $a = 10$, $b = 1$
  - (This leads to an SNR target of -18dB for voice and -11dB for data which are typical in UMTS.)
- Traffic demand density ρ composed of 90% of voice calls and 10% of data calls
- Plot $\tilde{P}$ as function of R for different values of ρ

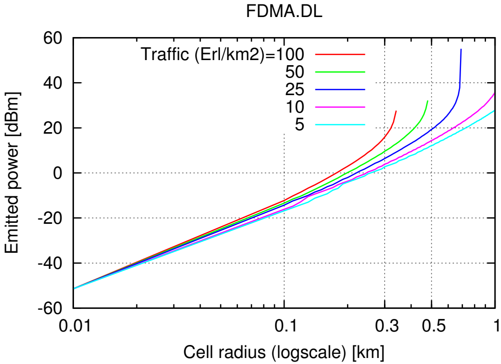
Figures

- Each curve has vertical asymptote at $R_{\infty}(\rho)$
- Curves for the different ρ are close and become linear when R becomes small : *coverage condition*.



Figures

- Densification brings the system from an interference limited regime (sensitivity to traffic density) to a *noise limited regime* (insensitivity to traffic density).



Conclusion

- Cumulative power received from all the interfering BS in a *large* hexagonal cellular network
  - Lower bound of cell radius above which the *safety zone* has not to be increased due to this cumulative effect
  - For propagation exponent $\beta \geq 3$, cell radius $28, 140, 280$m respectively for urban, suburban and rural environments respectively
- *Blocking probability* function of emitted power, cell radius and traffic demand density
  - Fixing some blocking probability target, calculate minimal emitted power required for a given configuration of the network
- Minimal emitted power as function of cell radius for a given traffic demand density :
  - permits to see whether the operator may *reduce* the power emitted currently in some parts of his network without degrading the QoS
  - Interesting in the perspective of a potential reduction of the regulatory exposure threshold

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