Electromagnetic Exposure and Quality of Service in the Downlink of Wireless Cellular Networks

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Paper [2]
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Introduction

• Study both
  • exposure of the persons to electromagnetic waves emitted by the base stations (BS)
  • quality of service (QoS) perceived by the users

• In wireless cellular networks
  • Either CDMA networks such as UMTS
  • or OFDMA networks such as LTE

• Safety distance:
  • already known when radiation from serving BS is taken into account
  • impact of interfering BS in a large (eventually infinite) network has not already been studied
  • First objective: study this cumulative effect and see whether the safety distance has to be increased

• Second objective: quantify the minimal emitted power required to assure a given QoS for a given traffic demand density and cell radius
Electromagnetic exposure

- Transition between near and far fields occurs at *Fraunhofer distance*

\[ r_0 = \frac{2D^2}{\lambda} \]

where \( \lambda \) wavelength, \( D \) antenna largest dimension

- Typically \( D \simeq 8\lambda \), then for a carrier frequency 2GHz

\[ r_0 \simeq 19.2m \]

- We aim to study radiation caused by the *interfering* BS (i.e. all the BS other than the serving one)

- Assume that inter-BS distance larger than \( 2r_0 \), then the cell of a given BS is in the far-field of other BS
Exposure limit

- Exposure in the *far-field* expressed in terms of *electric field strength* $E$
- Power $P_G$ *at the output* of a receiving antenna with gain $G$

$$P_G = \frac{\lambda^2 G E^2}{4\pi \eta_0}$$

where $P_G$ in W, $E$ in V/m, $\lambda$ wavelength in m and $\eta_0 = 377\Omega$

- Power $p$ *at the input of the receiver*

$$p := \frac{P_G}{G} = \frac{\lambda^2 E^2}{4\pi \eta_0}$$

called *received power*.

- Let $p_t$ and $E_t$ be the maximum admissible received power and electric field strength respectively

$$p_t = \frac{\lambda^2 E_t^2}{4\pi \eta_0}$$
Propagation loss

- If a BS transmits a power $\tilde{P}$, then received power at a distance $r$

\[
p(r) = \frac{\tilde{P}}{L(r)}
\]

where $L(r)$ is the propagation loss

- Formulae for propagation loss due to distance are well known for the far field region. Worst situation for people exposure is free space

\[
L_0(r) = (K_0 r)^{\beta_0}, \quad r \geq r_0
\]

where $\beta_0 = 2$ and $K_0 = \frac{4 \pi}{\lambda}$

- If obstacles, then the free space model under-estimates the propagation-loss. In this case, it is usual to take

\[
L(r) = (K r)^{\beta}, \quad r \geq r_0
\]

where $\beta > 2$ and $K > 0$, as for example the Hata model [1]
Radiation caused by *interfering* BS

- For \( m \) served by \( u \), power received from interfering BS

\[
g(m) = \sum_{v \neq u} \frac{\tilde{P}}{L_{v,m}}, \quad m \in u
\]

where \( L_{v,m} \) loss between \( v \) and \( m \), \( m \in u \) means \( m \) served by \( u \)

- BS on *infinite hexagonal* grid. For a given BS \( u \), the other BS are located on hexagons \( \mathcal{L}_1, \mathcal{L}_2, \ldots \)

\[
g(m) = \sum_{k \geq 1} g_k(m), \quad \text{where } g_k(m) := \sum_{v \in \mathcal{L}_k} \frac{\tilde{P}}{L_{v,m}}
\]
Radiation caused by interfering BS

- Contribution of BS in $L_k$ in terms of $L_1$

\[
g_k(m) = \sum_{l=0}^{k-1} (k^2 + l^2 - kl)^{-\beta/2} \frac{me^{-\frac{i l \pi}{3k}}}{\sqrt{k^2 + l^2 - kl}}
\]

- Bounds

\[
\zeta(\beta - 1) \inf_{n \in u} g_1(n) \leq g(m) \leq \left( \frac{2}{\sqrt{3}} \right)^\beta \zeta(\beta - 1) \sup_{n \in u} g_1(n)
\]

where $\zeta(x) = \sum_{k=1}^{\infty} k^{-x}$ is the Riemann zeta function

- Let $\Delta$ be the inter-BS distance, then

\[
g(m) \leq \left( \frac{2}{\sqrt{3}} \right)^\beta \frac{6\zeta(\beta - 1) \tilde{P}}{L(\Delta/2)}
\]
Radiation caused by *interfering* BS

- We deduce a condition assuring that $g(m)$ is less than some proportion, say $\delta$, of the maximum admissible received power $p_t$
- For a given constant $\delta > 0$, if

$$
\Delta \geq \frac{4}{K \sqrt{3}} \left( \frac{6\zeta (\beta - 1) \tilde{P}}{\delta p_t} \right)^{1/\beta}
$$

then $g(m) \leq \delta p_t$
- If $\delta$ sufficiently small (typically $\delta = 0.01$), the above inequality gives the inter-BS distance above which the effect of interfering BS may be neglected when evaluating the *safety zone*
Model specification

- ICNIRP recommends $E_t = 61\, \text{V/m}$, thus the maximum admissible received power

$$p_t = 0.022\, \text{W} \quad (= 13.5\, \text{dBm})$$

- Propagation parameters $\beta \in [2, 5]$ and $K = 10, 2, 1\, \text{m}^{-1}$ corresponding to urban, suburban and rural environments respectively

- Antennas gain 12dBi; BS transmit a power 43dBm; thus $\tilde{P} = 43 + 12 = 55\, \text{dBm} \quad (= 316\, \text{W})$ when we account for antenna gain

- We plot cell radius (defined as $\Delta/2$) as function of $\beta$ such that $g \leq 0.01p_t$
Cell radius such that $g \leq 0.01p_t$

- Cell radius decreases with $\beta$
- For $\beta \geq 3$ and $K = 10, 2, 1m^{-1}$ we get cell radii 28, 140, 280m respectively
Model description

- Propagation loss depends only on the distance (no shadowing)
- Each BS transmits a given *power* and a constant power spectral density
- Cells *don’t evolve in time*
- Users perform *single user detection*; the interference is considered as noise
- Bit-rate $r$ related to bandwidth $w$ and SINR

\[ r \leq bw \log_2 \left( 1 + \frac{1}{a} \text{SINR} \right) \]  

(1)

- Real-time calls require to be served at a given bit-rate for some duration
- *Blocking probability* as a measure of the QoS
Admission condition

- We will show that a suitable admission condition has the multi-Erlang form

\[ \sum_{m \in u} \varphi(m) \leq 1 \] (2)

for each BS \( u \); where \( \varphi(m) \) is some function

- In order to express \( \varphi(m) \), we introduce the interference factor

\[ f(m) = \sum_{v \neq u} \frac{L_{u,m}}{L_{v,m}}, \quad m \in u. \]

and the modified version of the interference factor

\[ \hat{f}(m) = \frac{1}{1 - \epsilon} \left( \frac{N L_{u,m}}{P} + \alpha + f(m) \right), \quad m \in u \]

\( N \) noise power, \( \alpha \) orthogonality factor, \( \epsilon \) fraction of power used by common channels
CDMA

- SINR of user $m \in u$

$$\text{SINR}_m = \frac{P_{u,m}/L_{u,m}}{N + \alpha (\tilde{P}_u - P_{u,m})/L_{u,m} + \sum_{v \neq u} \tilde{P}_v/L_{v,m}}$$

$P_{u,m}$ power allocated to user $m$.

- Combining the above equation with link constraint (1)

$$P_{u,m} \geq (1 - \epsilon) \tilde{P}_u \hat{f}(m) \frac{\xi_m}{1 + \alpha \xi_m}, \quad m \in u$$

where

$$\xi_m := a \left(2^{r_m/bW} - 1\right)$$

- Such power allocation exists iff (2) holds true with

$$\varphi(m) = \hat{f}(m) \frac{\xi_m}{1 + \alpha \xi_m}$$
OFDMA

- Each BS transmits a constant power spectral density
  \[ P_{u,m} = \frac{w_m}{W} (1 - \epsilon) \hat{P}_u, \quad m \in u, u \in U \]

- SINR of user \( m \in u \)
  \[ \text{SINR}_m = \frac{P_{u,m}/L_{u,m}}{\frac{w_m}{W} N + \frac{w_m}{W} \sum_{v \neq u} \tilde{P}_v/L_{v,m}} = \frac{1}{\hat{f}(m)} \]

- Combining the above equation with link constraint (1)
  \[ r_m \leq bw_m \log_2 \left( 1 + \frac{1}{a \hat{f}(m)} \right), \quad m \in u \]

- Such bandwidth allocation \((w_m)_{m \in u}\) exists iff (2) holds true with
  \[ \varphi(m) = \frac{r_m}{bW \log_2 \left[ 1 + 1/\left( a \hat{f}(m) \right) \right]} \]
Blocking probability evaluation

- BS on a regular hexagonal grid. Network is decomposed into $J$ bins of surfaces $s_j$, $j = 1, \ldots, J$
- Real-time calls whose inter-arrival times to bin $j$ are i.i.d. exponential r.v. with rate $\lambda_j$
- Durations of different calls are i.i.d. exponentially distributed with mean $1/\mu$
- $\rho$ defined by

$$\rho_j = \frac{\lambda_j}{\mu s_j}, \quad j = 1, \ldots, J$$

is the traffic demand density

- We evaluate the blocking probability by using the Kaufman-Roberts algorithm
Power versus radius

- blocking probability $b(R, \rho, \tilde{P})$ function of cell radius $R$, traffic demand density $\rho$ and emitted power $\tilde{P}$
- For a given blocking probability threshold $b_0$, solving

$$b(R, \rho, \tilde{P}) = b_0$$

we get an implicit expression of $\tilde{P}$ as function of $R$ and $\rho$
- We present two observations which are useful to interpret the numerical results below.
Power lower bound

- If a unique user $m$ in cell $u$, then (2) implies

$$\varphi(m) \leq 1, \quad m \in u$$

which may be seen as a coverage condition

- It is then natural to require that the above condition always holds true

- Using the expressions of $\varphi(m)$, the above condition writes

$$\tilde{P} \geq \frac{N L_{u,m}}{(1 - \epsilon) / \hat{\xi}_m - \alpha - f(m)}.$$  

Thus the emitted power is lower bounded

- This lower bound depends on the cell radius but not on the traffic demand
Cell radius upper bound

- Consider the theoretical case
  \[ \tilde{P} = \infty \]
  called usually the *pole* point

- Then solving
  \[ b(R, \rho, \infty) = b_0 \]
  we may view \( R \) as an implicit function of \( \rho \)

- Denote this function by \( R_\infty(\rho) \)
Model specification

- System bandwidth $W = 5$ MHz; common channel power is a fraction $\epsilon = 0.12$ of $\tilde{P}$; noise power $N = -103$ dBm

- Real-time calls which may be either voice at 12.2 Kbits/s or data at 64 Kbits/s

- Account for fading in link performance formula (1)
  $a = 10$, $b = 1$
  - (This leads to an SNR target of $-18$ dB for voice and $-11$ dB for data which are typical in UMTS.)

- Traffic demand density $\rho$ composed of 90% of voice calls and 10% of data calls

- Plot $\tilde{P}$ as function of $R$ for different values of $\rho$
Figures

- Each curve has vertical asymptote at $R_\infty (\rho)$
- Curves for the different $\rho$ are close and become linear when $R$ becomes small: coverage condition.
Figures

- Densification brings the system from an interference limited regime (sensitivity to traffic density) to a *noise limited regime* (insensitivity to traffic density).
Conclusion

- Cumulative power received from all the interfering BS in a large hexagonal cellular network
  - Lower bound of cell radius above which the safety zone has not to be increased due to this cumulative effect
  - For propagation exponent $\beta \geq 3$, cell radius 28, 140, 280 m respectively for urban, suburban and rural environments respectively

- *Blocking probability* function of emitted power, cell radius and traffic demand density
  - Fixing some blocking probability target, calculate minimal emitted power required for a given configuration of the network

- Minimal emitted power as function of cell radius for a given traffic demand density:
  - permits to see whether the operator may *reduce* the power emitted currently in some parts of his network without degrading the QoS
  - Interesting in the perspective of a potential reduction of the regulatory exposure threshold
Bibliography

**COST 231.**
*Final report: Digital mobile radio towards future generation systems.*

**M. K. Karray.**
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