Evaluation of the Blocking Probability and the Throughput in the Uplink of Wireless Cellular Networks

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***Abstract*—The objective of the present paper is to propose a rapid and accurate method for the *evaluation* of the quality of service (QoS) perceived by the users in the *uplink* of wireless cellular networks. In doing so we aim to account for the *dynamics* induced by the arrivals and the departures of the users. In particular, the QoS is evaluated in terms of the *blocking probability* for streaming users and the *throughput* of elastic calls.**

We build some conditions of the feasibility of the resource (power and bandwidth) allocation problem. We first develop a *reference feasibility condition* (FC) for which the QoS can be evaluated only by long simulations. Then we propose a *sufficient feasibility condition* (SFC) and an *analytical* method to evaluate the corresponding QoS. The blocking probability for streaming users is evaluated using the *Kaufman-Roberts algorithm* whereas the *throughput* of elastic calls is evaluated using a *multi-class processor sharing model*.

The proposed approach is validated by simulating a CDMA network as well as an OFDMA one.

***Index Terms*—Cellular, Wireless, OFDMA, CDMA, QoS, Uplink, Blocking probability, Throughput.**

I. Introduction

This paper aims at developing a method to evaluate the *quality of service* (QoS) perceived by the users in the *uplink* of wireless cellular networks. In doing so, we aim to take into account the *dynamics* of the calls arrivals and departures and the evolution of the users locations induced by these dynamics.

We consider both Code-Division Multiple Access (CDMA)[1] and Orthogonal Frequency-Division Multiple Access (OFDMA)[2] cellular networks. Such networks may serve either *streaming* (i.e., real time) traffic or *elastic* (i.e., nonreal time) traffic. Each streaming user requires to be served at a given bit-rate for a given duration, whereas an elastic call requires to transmit some data volume at a bit-rate which may be decided by the network. In order to account for the dynamics of the calls arrivals and departures, the QoS perceived by the users is evaluated in terms of the *blocking probability* of streaming calls and the *throughput* or delay for elastic ones. Evaluation of these QoS metrics is particularly important for the network *dimensioning*; i.e., evaluating the minimal number of base stations assuring some QoS (for some given traffic demand). This permits to minimize the network cost.

A. Outline of our approach

We begin by formulating the *resource* (*power* and *bandwidth*) allocation problem in the considered networks. In doing so we account for the power and bandwidth limitations as well as for the single link performance with the help of information theory. Then we establish a *reference feasibility condition*, denoted FC; i.e. a necessary and sufficient condition for the resource allocation problem to admit a solution.

Then we account for the dynamics induced by the call arrivals and departures. In this context, a natural idea is to admit a new user to the network only when FC (with this new user) is respected. Unfortunately, in this case the users QoS (i.e. blocking probability, throughput) may be evaluated only by long simulations. This is due to the fact that FC has not the *multi-Erlang form*. (A condition is said to have the *multi-Erlang form* if it can be written as the weighted sum of the bit-rates of the users not exceeding some constant.).

In order to cope with this difficulty, we propose a *sufficient feasibility condition* (SFC) of the resource allocation problem. SFC assures that the resource allocation is feasible, but may sometimes block some arrivals even though the resource allocation is feasible. This induces a *loss of capacity* compared to the reference FC which we shall evaluate by simulations and show that it remains moderate. Unfortunately, even though SFC is simpler than FC, it has not the multi-Erlang form. We shall then propose an analytical approximate method to evaluate the users QoS for SFC. This approximation shall be validated by comparing its results to those obtained by simulations.

The proposed approach permits a rapid and accurate evaluation of the blocking probability using the *Kaufman-Roberts algorithm* [1], [2]. Moreover, the throughput of the elastic

[1] Typical examples: Universal Mobile Telecommunications System (UMTS); High Speed Uplink Packet Access (HSUPA).

[2] Typical examples: 3GPP Long Term Evolution (LTE) system; IEEE 802.16 WirelessMAN Air Interface standard (WiMAX).

traffic is calculated by using a *multi-class processor sharing model*. These tools are in the field of *queueing theory*.

B. Paper organization

The remaining part of this paper is organized as follows. In the next subsection we discuss briefly the related work. We present our model assumptions and give the basic notations in Section II. We describe the resource allocation problem, give conditions for its feasibility and build analytical methods for QoS evaluation in Sections III and IV for CDMA and OFDMA respectively. The validation of the proposed approach is described in Section V.

C. Related works

The *uplink* of CDMA networks is studied by the authors of [3] who propose a method to approximate the so-called *outage probability*, which is the probability that the signal-to-interference ratio is less than some threshold, when users, modeled as a Poisson point process, are all accepted. It may be easily seen from [4] for example that the outage probability is different from the blocking probability which is the suitable indicator of the QoS perceived by the users in cellular networks. An attempt to propose a method to calculate the blocking probability for the uplink of CDMA networks is made in [5] and [6]. Unfortunately only heuristic arguments are given there. In particular, the derived methods are not compared to a reference one and they are not clearly related to the resource allocation problem (and its feasibility). Staehle et al. [7] derives a method to calculate the interference in the uplink of CDMA networks accounting for the random locations of the users, but this paper doesn't address the problem of evaluating the blocking probability. It is shown in [8] that in some cases the uplink of CDMA networks may oscillate between some states with a significant sojourn time in each state (a phenomenon called *metastability*). In the present work we shall consider the average of the blocking probability over a time interval sufficiently large so that the *ergodic* averages over the different states are obtained.

The problem of resource allocation in the uplink of OFDMA networks is studied in several papers such as [9] but such work doesn't lead to a simple QoS evaluation method in a dynamic context. An attempt to propose such method for OFDMA is made for example in [10] and [11], but as for CDMA case, only heuristic arguments are given and the derived method is not compared to a reference one.

Epstein and Schwartz [12] study the QoS in terms of blocking and dropping probabilities, but the interference between the users is not taken into account explicitely. In particular it is not shown how to evaluate the coefficients $W_1, \ldots, W_K$ pondering the number of users in the admission condition in [12, Eqs (12), (13)]. The effect of interference on power allocation feasibility and on admission control is studied for example in [13] but only the downlink is studied there. Hou and Kumar [14] study the QoS in wireless local area and sensor networks whereas we focus on cellular networks in the present study.

The present work adopts the approach of [15] and [16, §3.2] which build feasibility conditions for the resource allocation for the uplink of CDMA networks. We shall propose in the present paper a new efficient method for the evaluation of QoS in CDMA networks and extend the whole approach to the uplink of OFDMA networks. An analogous work for the downlink is reported in [17] and [18].

II. BASICS

A. Model assumptions

We will consider a wireless network composed of several base stations (BSs) serving some users. User's *power is limited* to some given maximal value. The same frequency spectrum is available to all BSs (*frequency reuse* factor equal to *one*). There is no macro-diversity; i.e., each user is served by exactly one BS. We assume that each BS serves users is some exclusive geometric cell associated to it, which *does not evolve in time*.

Users perform *single user detection*; the interference is considered as noise. The bit-rate r of a given user is related to its bandwidth w and its SINR (signal to interference and noise power ratio) by

$$r \leq bw \log_2 \left(1 + \frac{1}{a} \mathrm{SINR} \right) \tag{1}$$

where the constants a and b permit to account for the loss in practical systems compared to the ideal AWGN case (for which the above formula applies with $a = b = 1$). A value $b < 1$ may account for the loss of bandwidth due to signalling [19] and we may take $a \neq 1$ to account for the effect of fading on capacity as shown in [20].

1) Traffic: We assume that the *users don't move* during their service. Streaming calls arrive to the network and require to be served at a given bit-rate for some duration. The streaming traffic demand (expressed in Erlang) is defined as the ratio between the mean call duration and the mean inter-arrival time. If the network isn't able to satisfy the new arriving user together with the existing ones, then the new call is blocked. We shall consider the *blocking probability* as a measure of the QoS perceived by the streaming users.

Elastic calls require at their arrival to transmit some volume of data. The elastic traffic demand (expressed in bits per second) is defined as the ratio between the mean data volume and the mean inter-arrival time. The network may modify the bit-rates of all the elastic users when necessary. Therefore, in the long run of the system, each user gets some average *throughput* (or equivalently, transmits its data within some duration called *delay*). We shall consider this average throughput as the measure of the QoS perceived by the elastic users.

Remark 1: For elastic traffic, when the traffic demand exceeds some *critical* value, the throughput of each user goes to zero. (A typical example of this situation is the $\mathrm{M/M/1}$ queue when the traffic demand exceeds 1.) This *unstable* situation has to be avoided. Thus a second key QoS metric for elastic traffic is the *critical traffic demand*.

2) Interference in OFDMA networks: In OFDMA networks, the whole spectrum is divided into sub-carriers. Each base station allocates disjoint sub-carriers to its users. Thus, any given user receives interference only from users in other base stations. This interference equals the sum of powers emitted by other BS users on the sub-carriers allocated to him by his BS. We shall assume that the interference power spectral density is constant in the whole spectrum. This assumption may be justified by the law of large number if the number of (strong) interfering users is large enough. If this is not the case, a suitable fast sub-carrier permutation (for a given configuration of users) may give a further justification of this assumption.

B. Notation

The network is composed of a finite set U of BS located on the plane. Lets denote by $L_{u,m}$ the propagation-loss between a BS u and a given user m (not including the fading). We will write $m \in u$ when user m is served by base station u. Each user m transmits a power P_m which may not exceed a maximal value $\tilde{P}_m$; that is $P_m \leq \tilde{P}_m$. Let N_0 be the power spectral density of external noise, W be the system bandwidth and $N = WN_0$ be the noise power. The total power (sum of noise and powers from all the users) received at BS u is

$$I_u := N + \sum_{v \in U} \alpha_{u,v} \sum_{n \in v} P_n / L_{u,n} \tag{2}$$

where $\alpha_{u,v} = 1$ for OFDMA and $\alpha_{u,v} = 1_{\{u=v\}}$ for CDMA.

In order to simplify the formulae in the remaining part of the paper, we introduce the following notation (its relevance will become clear later). For a user m with bit-rate r_m and bandwidth w_m, we introduce

$$\xi_m := \frac{a w_m}{W} \left(2^{\frac{r_m}{b w_m}} - 1 \right)$$

or equivalently

$$r_m = b w_m \log_2 \left(1 + \frac{W}{a w_m} \xi_m \right) \tag{3}$$

which shows that ξ_m is closely related to the SINR in Shannon's formula. It will be useful to introduce the following modification of ξ_m:

$$\hat{\xi}_m = \xi_m / \left(1 + \alpha \xi_m\right). \tag{4}$$

where $\alpha = 1$ for OFDMA and $\alpha = 0$ for CDMA.

Finally, the so-called *f-factor* (or *interference factor*) is defined by

$$f(m) = \sum_{v \neq u} \frac{L_{u,m}}{L_{v,m}}, \quad m \in u. \tag{5}$$

III. QoS EVALUATION OF CDMA

In a CDMA system, each user is allocated all the bandwidth; that is $w_m = W$. The SINR of a user m served by a BS u is equal to

$$\mathrm{SINR}_m = \frac{P_m / L_{u,m}}{N + \sum_{v \in U} \alpha_{u,v} \sum_{n \in v \setminus \{m\}} P_n / L_{u,n}}$$

Using (1) we deduce that the *resource allocation problem* consists of looking for powers (P_m) and bit-rates (r_m) such that for all users $m \in u$ and all BS $u \in U$

$$\begin{cases} r_m \leq bW \log_2 \left(1 + \frac{1}{a} \frac{P_m / L_{u,m}}{N + \sum_{v \in U} \alpha_{u,v} \sum_{n \in v \setminus \{m\}} P_n / L_{u,n}} \right) \\ P_m \leq \tilde{P}_m \end{cases} \tag{6}$$

which are the information theoretic and the maximal-power constraints respectively. All the powers and the bit-rates should be nonnegative which will be implicit in the formulations of our problems.

A. Reference feasibility condition (FC)

Definition 1: We will say that a vector of user bit-rates (r_m) is *feasible* if there exist powers (P_m) such that the constraints in (6) are satisfied. In this case we will also say that (r_m) satisfies the *(reference) feasibility condition* (FC).

An interesting idea is to use FC as admission control scheme. The network admits a new streaming call when its bit-rate together with those of currently served users satisfy FC. Unfortunately, in this case the user's QoS (blocking probability, throughput) may be evaluated only by time consuming simulations. This is due to the fact that FC has not the *multi-Erlang form*; i.e., FC can not be written as the weighted sum over some classes of the number of users in each class less than some constant. We shall build in the next section a condition assuring the feasibility of Problem (6) and having the multi-Erlang form. This particular form will permit an analytical evaluation of the QoS metrics in the subsequent sections.

B. Sufficient feasibility condition (SFC)

We begin by expressing the Problem (6) in terms of the interference vector $I = (I_u)_{u \in U}$ where I_u is the *total interference* received at base station u defined by (2).

Proposition 1: Problem (6) is feasible (i.e., admits a solution) iff the following problem

$$\begin{cases} (\mathbf{1} - B)\, I \geq \mathbf{N} \\ I \leq \tilde{I} \end{cases} \tag{7}$$

is feasible; where the matrix $B = [B_{u,v}]_{u,v \in U}$ is defined by

$$B_{u,v} = \alpha_{u,v} \sum_{n \in v} L_{v,n} / L_{u,n} \hat{\xi}_n \tag{8}$$

the vector $\mathbf{N} = (\mathbf{N}_u)_{u \in U}$ is defined by

$$\mathbf{N}_u = N \tag{9}$$

and the vector $\tilde{I} = \left(\tilde{I}_u\right)_{u \in U}$ is defined by

$$\tilde{I}_u = \inf_{m \in u} \frac{\tilde{P}_m}{\hat{\xi}_m L_{u,m}} \tag{10}$$

where $\hat{\xi}_m$ is given by (4). Moreover in case of feasiblity of (6), the vector of powers defined by

$$P_m = \hat{\xi}_m L_{u,m} I_u, \quad m \in u \in U \tag{11}$$

is solution.

Proof: First note that Problem (6) is equivalent to

$$\left\{ \begin{array}{l} \frac{P_m/L_{u,m}}{N+\sum_{v\in U}\alpha_{u,v}\sum_{n\in v}P_n/L_{u,n}} \geq \hat{\xi}_m, \\ P_m \leq \tilde{P}_m, \end{array} \right. \tag{12}$$

for all $m \in u$ and all $u \in U$.

Direct. Assume that (12) is feasible. Let $I = (I_u)_{u\in U}$ be defined by (2). Then the first inequality in (12) implies

$$P_m \geq \hat{\xi}_m L_{u,m} I_u, \quad m \in u \in U \tag{13}$$

Thus we have for all $n \in v \in U$, $P_n \geq \hat{\xi}_n L_{v,n} I_v$. Multiplying this inequality by $\alpha_{u,v}/L_{u,n}$ and adding over $n \in v \in U$, we get

$$I_u - N \geq \sum_{v\in U}\alpha_{u,v}\sum_{n\in v}\hat{\xi}_n \frac{L_{v,n}}{L_{u,n}} I_v = \sum_{v\in U} B_{u,v} I_v.$$

Thus $I = (I_u)_{u\in U}$ satisfies the first inequality in (7). On the other hand (13) and the second inequality in (12) imply

$$\tilde{P}_m \geq \hat{\xi}_m L_{u,m} I_u, \quad m \in u \in U$$

which implies $I_u \leq \tilde{I}_u$ for all $u \in U$. Thus (7) is feasible.

Reverse. Inversely, assume now that (7) is feasible. Let $P = (P_m)_{m\in u\in U}$ be defined by (11). The first inequality in (7) implies

$$I_u \geq N + \sum_{v\in U}\alpha_{u,v}\sum_{n\in v}\hat{\xi}_n \frac{L_{v,n}}{L_{u,n}} I_v$$

which combined with (11) gives

$$\frac{P_m}{\hat{\xi}_m L_{u,m}} \geq N + \sum_{v\in U}\alpha_{u,v}\sum_{n\in v} P_n/L_{u,n}$$

which implies the first inequality in (12). On the other hand (11) and the second inequality in (7) imply $P_m \leq \tilde{P}_m$. Thus (12) is feasible. ■

Using the above proposition and the Perron-Frobenius theorem [21, p.670], one may express the reference feasibility condition FC as follows

$$\left\{ \begin{array}{l} \boldsymbol{\rho}(B) < 1 \\ (\mathbf{1} - B)^{-1}\mathbf{N} \leq \tilde{I} \end{array} \right.$$

which has not the multi-Erlang form. We now give a sufficient feasibility condition for Problem (6).

Proposition 2: Assume that there exists some non-negative vector $J = (J_u)_{u\in U}$ satisfying

$$J \leq \tilde{I} \tag{14}$$

(where $\tilde{I}$ is given by (10)) and

$$\sum_{v\in U}\alpha_{u,v} J_v \sum_{n\in v} L_{v,n}/L_{u,n}\hat{\xi}_n \leq J_u - N, \quad u \in U. \tag{15}$$

Then Problem (6) is feasible.

Proof: It is straightforward to check that (15) is equivalent to $(\mathbf{1} - B) J \geq \mathbf{N}$. The desired result is then immediate from Proposition 1. ■

Condition (15) where $J \leq \tilde{I}$ is called *sufficient feasibility condition* (abbreviated by SFC).

The following remark gives a physical interpretation of SFC.

Remark 2: Assume that $\hat{\xi}_m = \hat{\xi}$ is independent of the particular user m. In this case, we deduce from (11) that $P_m = \hat{\xi} L_{u,m} J_u$; thus $\frac{P_m}{L_{u,m}} = \hat{\xi} J_u$. That is SFC corresponds to the situation where *all the user are received at their serving BS with the same power*.

Unfortunately, SFC has not the *multi-Erlang form*. Thus the corresponding QoS (blocking probability, throughput) may only be evaluated by simulations. We shall derive an approximate analytic method in the next section.

C. Approximating SFC's QoS

1) Streaming traffic: We take $J_u = \tilde{I}_u$ which clearly satisfies (14). On the other hand, note that SFC (15) takes the form $J_u \sum_{m\in u}\hat{\xi}_m + I_u^{\mathrm{inter}} \leq J_u - N$; or equivalently

$$\sum_{m\in u}\hat{\xi}_m \leq 1 - \frac{N + I_u^{\mathrm{inter}}}{J_u} \tag{16}$$

where I_u^{inter} is defined as follows

$$I_u^{\mathrm{inter}} = \sum_{v\neq u} J_v \sum_{n\in v} L_{v,n}/L_{u,n}\hat{\xi}_n.$$

Remark 3: The parameter I_u^{inter} defined by the above equation may be interpreted as the *inter-cell interference*. Indeed, we deduce from (11), that the vector of powers associated to the interference vector J equals

$$P_m = \hat{\xi}_m L_{u,m} J_u, \quad m \in u$$

thus the corresponding *inter-cell interference* equals

$$\sum_{v\neq u}\sum_{n\in v} P_n/L_{u,n} = \sum_{v\neq u} J_v \sum_{n\in v} L_{v,n}/L_{u,n}\hat{\xi}_n = I_u^{\mathrm{inter}}.$$

a) Approximation of the inter-cell interference: We shall use an approximation for the inter-cell interference I_u^{inter} which is largely adopted in literature (see for example [22] and [3]). If all the BS play a symmetric role, then we may take $J_u = J$ independent of the particular cell u. In this case

$$I_u^{\mathrm{inter}} = J \sum_{v\neq u}\sum_{n\in v} L_{v,n}/L_{u,n}\hat{\xi}_n$$

which may be approximated by (using an argument of type 'law of large numbers')

$$I_u^{\mathrm{inter}} \simeq J \bar{M} \bar{\hat{\xi}} \bar{f}$$

where f is the f-factor defined in (5); $\bar{\hat{\xi}}$ and $\bar{f}$ are the averages of $\hat{\xi}$ and of f respectively over the cell; and $\bar{M}$ is the traffic demand per cell. Using the above approximation, (16) becomes

$$\sum_{m\in u}\hat{\xi}_m \leq 1 - \frac{N}{J} - \bar{\hat{\xi}}\bar{M}\bar{f}. \tag{17}$$

In the case of a single streaming class, i.e., $\hat{\xi}_m = \hat{\xi}$ is independent of the particular user m, the above equation becomes $M \leq \frac{1}{\hat{\xi}} - \frac{N}{\hat{\xi}J} - \bar{M}\bar{f}$ where M is the number of users in the cell. Then the blocking probability may be calculated by the famous Erlang's loss formula with a traffic demand $\bar{M}$ and a number of servers equal to $\frac{1}{\hat{\xi}} - \frac{N}{\hat{\xi}J} - \bar{M}\bar{f}$.

Consider now the more general case where there are multiple streaming classes (voice, streaming video, etc.). In this case $\hat{\xi}_m$ depends on the class of user m and the above condition has a multi-Erlang form. Thus the blocking probability may be easily evaluated by the Kaufman-Roberts algorithm [1], [2].

2) Elastic traffic: Assume that we allocate a bit-rate to each user in such a way that $\frac{\tilde{P}_m}{\hat{\xi}_m}$ is independent of the particular $m \in u$. We take

$$J_u = \frac{\tilde{P}_m}{\hat{\xi}_m \tilde{L}}, \quad m \in u$$

where

$$\tilde{L} := \sup_{n \in v, v \in U} L_{v,n} \tag{18}$$

Attempting to satisfy (15) with equality, we get after straightforward calculations

$$\hat{\xi}_m = \tilde{P}_m \left(N\tilde{L} + \sum_{v \in U} \alpha_{u,v} \sum_{n \in v} L_{v,n}/L_{u,n}\tilde{P}_n \right)^{-1}. \tag{19}$$

Using (3) and (4) we deduce that

$$r_m = bW \log_2 \left(1 + \frac{1}{a} \frac{1}{1/\hat{\xi}_m - \alpha} \right)$$

where $\hat{\xi}_m$ is given by (19). The above equation gives the bit-rates of the users in a given cell as function of the number and positions of the users in all the network. The corresponding QoS (in particular the average throughput per user) may be evaluated by simulations. We shall now propose an approximate analytical method for this evaluation.

a) Approximation: Consider the case where $\tilde{P}_m = \tilde{P}$ is independent of the particular m. Then (19) becomes

$$\hat{\xi}_m = \left(\frac{N\tilde{L}}{\tilde{P}} + \sum_{v \in U} \alpha_{u,v} \sum_{n \in v} L_{v,n}/L_{u,n} \right)^{-1}.$$

In order to approximate the QoS, we make the following heuristic (justified by an argument of type 'law of large numbers'). Replacing $\sum_{v \in U} \alpha_{u,v} \sum_{n \in v} L_{v,n}/L_{u,n}$ by $M_u \left(\alpha + \bar{f} \right)$ (where M_u is the number of users in the cell u) the above equation becomes

$$\hat{\xi}_m = \left(\frac{N\tilde{L}}{\tilde{P}} + \left(\alpha + \bar{f} \right) M_u \right)^{-1}.$$

Combining the above equation with (3), and (4) we get

$$r_m = bW \log_2 \left(1 + \frac{1}{a} \frac{1}{\frac{N\tilde{L}}{\tilde{P}} - \alpha + \left(\alpha + \bar{f} \right) M_u} \right) =: \frac{1}{h(M_u)} \tag{20}$$

where the function $h(M_u)$ is defined by the second equality of the above equation.

Proposition 3: For the allocation (20), the system is stable (see Remark 1 for the definition of this term) when the traffic demand per cell denoted ρ satisfies the following constraint

$$\rho < \lim_{n \to \infty} \frac{n}{h(n)} = \frac{bW}{a \left(\alpha + \bar{f} \right) \ln 2}.$$

If the system is stable, then at the steady state, the expected number of users in cell u, the *mean delay* and *throughput* per user are given respectively by

$$\bar{N} = \rho \mathcal{H}(\rho), \quad \bar{T} = \frac{\mathcal{H}(\rho)}{\mu}, \quad \bar{r} = \frac{1}{\mathcal{H}(\rho)} \tag{21}$$

where the function $\mathcal{H}(s)$ is defined for $s > 0$ by

$$\mathcal{H}(s) = \frac{\mathbf{E}[H(X+1)]}{\mathbf{E}[H(X)]}, \; H(M) = \begin{cases} \prod_{k=1}^{M} h(k) & \text{if } M \geq 1 \\ 1 & \text{if } M = 0 \end{cases} \tag{22}$$

where X is a Poisson random variable with parameter s.

Proof: The results for stability and the number of users $\bar{N}$ at the steady state follows from properties of generalized processor sharing queues [23, Proposition 3.1]. Applying Little's formula [24] we get the desired result for the delay $\bar{T}$. Recalling that the throughput $\bar{r}$ is the ratio of the date volume average $1/\mu$ and the delay $\bar{T}$, finishes the proof. ■

IV. QoS EVALUATION OF OFDMA

In OFDMA networks, each base station u allocates some number of sub-carriers of the total width w_m from the total spectrum of width W to each user $m \in u$, in such a way that two different users of the same BS have disjoint subsets of sub-carriers. However, since the same frequency spectrum is allocated (assumed on average uniformly) by all base stations, user $m \in u$ receives interference from users in each cell $v \neq u$ of power $\frac{w_m}{W} \sum_{n \in v} \frac{P_n}{L_{u,n}}$. We assume that this interference acts as AWGN. The SINR of user $m \in u$ is equal to

$$\text{SINR}_m = \frac{P_m/L_{u,m}}{w_m N_0 + \frac{w_m}{W} \sum_{n \in v \neq u} P_n/L_{u,n}}$$

where N_0 is the power spectral density of the thermal noise; and where the notation $\sum_{n \in v \neq u}$ means $\sum_{v \in U \setminus \{u\}} \sum_{n \in v}$. Using (1) we deduce that the *resource allocation problem* in OFDMA may be formulated as follows. Find bandwidths (w_m), powers (P_m), and bit-rates (r_m) such that for all $m \in u$ and all $u \in U$

$$\begin{cases} r_m \leq bw_m \log_2 \left(1 + \frac{1}{a} \frac{P_m/L_{u,m}}{w_m N_0 + \frac{w_m}{W} \sum_{n \in v \neq u} P_n/L_{u,n}} \right) \\ P_m \leq \tilde{P}_m \\ \sum_{m \in u} w_m \leq W \end{cases} \tag{23}$$

which are the information theoretic, maximal-power and total-bandwidth constraints respectively.

A. Reference feasibility condition (FC)

Definition 2: We will say that a vector of user bit-rates (r_m) is *feasible* if there exist powers (P_m) and bandwidths (w_m) such that the constraints in (23) are satisfied. In this case we will also say that (r_m) satisfies the *(reference) feasibility condition* (FC).

As for CDMA, the QoS (blocking probability, throughput) evaluation is intractable analytically when FC is used as control scheme. We shall look in the subsequent sections for sufficient feasibility conditions permitting analytical calculus of the QoS.

B. Sufficient feasibility condition (SFC)

In order to get a sufficient feasibility condition, we begin, as for CDMA, by expressing Problem (23) in terms of the interference vector $I = (I_u)_{u\in U}$ where I_u is given by (2).

Proposition 4: Problem (23) is feasible iff the following problem

$$\begin{cases} (\mathbf{1} - B)\, I \geq \mathbf{N} \\ I \leq \tilde{I} \\ \sum_{m\in u} w_m \leq W, \quad u \in U \end{cases} \tag{24}$$

is feasible; where the matrix $B = [B_{u,v}]_{u,v\in U}$, the vector $\mathbf{N} = (\mathbf{N}_u)_{u\in U}$ and the vector $\tilde{I} = \left(\tilde{I}_u\right)_{u\in U}$ are defined by (8), (9) and (10) respectively.

Proof: Problem (23) is equivalent to

$$\begin{cases} \frac{P_m/L_{u,m}}{N+\sum_{v\in U}\alpha_{u,v}\sum_{n\in v}P_n/L_{u,n}} \geq \hat{\xi}_m \\ P_m \leq \tilde{P}_m \\ \sum_{m\in u} w_m \leq W \end{cases} \tag{25}$$

for all $m \in u$ and all $u \in U$. Note that the two first equations of (25) have the same form as (12). The rest of the proof relies then on Proposition 1. ■

Proposition 5: If there exists some non-negative vector $J = (J_u)_{u\in U}$ such that

$$J \leq \tilde{I}$$

and for all $u \in U$

$$\begin{cases} \sum_{v\in U}\alpha_{u,v}\sum_{n\in v}L_{v,n}/L_{u,n}\hat{\xi}_n J_v \leq J_u - N \\ \sum_{m\in u} w_m \leq W \end{cases} \tag{26}$$

then Problem (23) is feasible.

Proof: Note that the first equation in (26) is equivalent to $(\mathbf{1} - B)\, J \geq \mathbf{N}$. The desired result is then immediate from Proposition 4. ■

Condition (26) where $J \leq \tilde{I}$ is called *sufficient feasibility condition* (abbreviated by SFC). We shall now propose an approximate analytical method to evaluate its QoS.

C. Approximating SFC's QoS

Assume that we allocate a bandwidth (and a bit-rate when possible) to each user in such a way that $\frac{\tilde{P}_m}{\hat{\xi}_m}$ is independent of the particular $m \in u$. We take

$$J_u = \frac{1}{\tilde{L}}\frac{\tilde{P}_m}{\hat{\xi}_m}, \quad m \in u.$$

where $\tilde{L}$ is defined by (18). Combining the above equation with (26) we get

$$\hat{\xi}_m \leq \frac{\tilde{P}_m}{N\tilde{L} + \sum_{n\in v\neq u}\tilde{P}_n L_{v,n}/L_{u,n}}. \tag{27}$$

1) Streaming traffic: Recall that for streaming calls, the bit-rates of the users are pre-defined (fixed). We deduce from (27) that

$$w_m \leq \hat{\xi}_m^{-1}\left(\frac{\tilde{P}_m}{N\tilde{L} + \sum_{n\in v\neq u}\tilde{P}_n L_{v,n}/L_{u,n}}\right)$$

where $\hat{\xi}_m^{-1}$ is the inverse of the function $w \mapsto \hat{\xi}_m(w) = \frac{aw}{W}\left(2^{\frac{r_m}{bw}} - 1\right)$. From the second equation in (26), i.e. the bandwidth constraint $\sum_{m\in u} w_m \leq W$, we get

$$\sum_{m\in u}\hat{\xi}_m^{-1}\left(\frac{\tilde{P}_m}{N\tilde{L} + \sum_{n\in v\neq u}\tilde{P}_n L_{v,n}/L_{u,n}}\right) \leq W. \tag{28}$$

Analogously to CDMA, we make the following approximation $\sum_{v\neq u}\sum_{n\in v}\tilde{P}_n L_{v,n}/L_{u,n} \simeq \bar{\tilde{P}}\bar{M}\bar{f}$. Then the inequality (28) becomes

$$\sum_{m\in u}\hat{\xi}_m^{-1}\left(\frac{\tilde{P}_m}{N\tilde{L} + \bar{\tilde{P}}\bar{M}\bar{f}}\right) \leq W. \tag{29}$$

The above condition has a multi-Erlang form. Thus the corresponding blocking probability may be evaluated by the Kaufman-Roberts algorithm.

2) Elastic traffic: Assume that we allocate the same bandwidth to all the users in the same cell, that is $w_m = \frac{W}{M_u}$ for all $m \in u$.We shall attempt to satisfy (27) with equality, that is

$$\hat{\xi}_m = \frac{\tilde{P}_m}{N\tilde{L} + \sum_{v\neq u}\sum_{n\in v}\tilde{P}_n L_{v,n}/L_{u,n}}$$

Using (3), and (4) we get for $m \in u$

$$r_m = \frac{bW}{M_u}\log_2\left(1 + \frac{M_u}{a}\frac{\tilde{P}_m}{N\tilde{L} + \sum_{n\in v\neq u}\tilde{P}_n\frac{L_{v,n}}{L_{u,n}}}\right). \tag{30}$$

As for CDMA, the corresponding QoS may only be evaluated by simulations. We shall now propose an approximate analytical method.

a) Approximation: Consider the case where $\tilde{P}_m = \tilde{P}$ is independent of the particular m. Then for all $m \in u$

$$r_m = \frac{bW}{M_u}\log_2\left(1 + \frac{1}{a}\frac{M_u}{\frac{N\tilde{L}}{\tilde{P}} + \sum_{n\in v\neq u}\frac{L_{v,n}}{L_{u,n}}}\right).$$

In order to approximate the QoS, we approximate $\sum_{v\neq u}\sum_{n\in v}L_{v,n}/L_{u,n}$ by $M_u\bar{f}$. Then the above equation becomes

$$r_m = \frac{bW}{M_u}\log_2\left(1 + \frac{1}{a}\frac{M_u}{\frac{N\tilde{L}}{\tilde{P}} + \bar{f}M_u}\right) = \frac{1}{h(M_u)} \tag{31}$$

where the function $h(M_u)$ is defined by the second equality of the above equation.

Proposition 6: For the allocation (31), the system is stable when the traffic demand per cell

$$\rho < \lim_{n\to\infty}\frac{n}{h(n)} = bW\log_2\left(1 + \frac{1}{a\bar{f}}\right)$$

If the system is stable, then the QoS metrics are given by (21).

Proof: Analogous to the proof of Proposition 3. ■

Link\service	FC	SFC simul.	SFC approx.
CDMA	1day	3h	1s
OFDMA	1week	1day	1s

TABLE I
CALCULUS DURATIONS ON A TYPICAL PC.

V. VALIDATION OF THE SUFFICIENT CONDITION

A. Model specification

We consider the radio part of the uplink in wireless cellular networks. In order to obtain numerical values, we consider the most popular *hexagonal network model*, where the base stations are placed on a regular hexagonal grid. Let R be the radius of the disc whose area is equal to that of the hexagonal cell served by each base station, and call R the *cell radius*. We consider three values of the cell radius $R = 0.5$, 2 or 3km. We assume a path loss $L(r) = (Kr)^\eta$, with $\eta = 3.38$ and $K = 8667$ where r designates the distance between the transmitter and the receiver (no shadowing). Users arrive randomly (spatially) uniformly to the network and don't move during their calls.

The system bandwidth equals $W = 5$MHz. Base stations are equipped with omnidirectional antennas having a gain 12dBi and no loss. User maximal power equals 21dBm; thus $\tilde{P} = 21 + 12 = 33\text{dBm}$ when we account for antenna gain. The ambient noise power $N = WN_0 = -105\text{dBm}$.

Note that the numeric values of the parameters a and b in the link performance formula (1) don't alter fundamentally the form of FC and SFC. Therefore we take for the simulations $a = 1$ and $b = 1$; i.e. an AWGN channel. We assume that the network serves real-time calls with bit-rate 113Kbits/s (the corresponding SNR target equals -18dB).

B. Results

Figures 1 and 2 show the blocking probability as function of the traffic demand per cell for FC by simulations and for SFC either by simulations or approximation, for CDMA and OFDMA networks respectively. Table I gives the calculus durations. We see the important time saving due to SFC and particularly its analytic approximation. This is particularly useful for operators who aim to predict the QoS of their networks for several combinations of the parameters (for dimensioning, prediction or optimization).

We define the capacity as the traffic demand per cell corresponding to a blocking probability of 0.02 (typical in dimensioning). It is important to bound the loss of capacity induced by the sufficient feasibility conditions relatively to the reference FC. In particular, one may consider the naive condition which consists of blocking all the users. This is clearly a sufficient condition for the feasibility of the resource allocation, nevertheless it is far from efficient. We deduce from Figures 1 and 2 that SFC induces approximately 15% of loss of capacity compared to FC for both CDMA and OFDMA. This loss of capacity seems to be acceptable for network operators looking for rapid network dimensioning tools. Note also that it is evaluated with respect to the reference feasibility condition assuming some perfect control scheme. On the other hand, Figures 1 and 2 show that the capacity gap between SFC simulation and analytic approximation remains less than 4%. We conclude that the proposed QoS evaluation method (based on the SFC analytic approximation) is rapid and sufficiently accurate for the dimensioning process.

We have also tested numerically the situation when *all the users transmit at their maximal powers*; which is considered in [8]. This gives a dramatic decrease of capacity (up to 100%), thus this situation has to be avoided both in the networks, and in building QoS evaluation methods. Finally, our approach has the following advantages compared to the preceding methods: (i) there is no longer need to separate coverage and capacity problems; (ii) there is no need of a load factor nor a load threshold as in [10], [11]; and (iii) CDMA and OFDMA networks are treated in a coherent way.

VI. CONCLUSION

We develop rapid and accurate methods to evaluate the QoS perceived by the users in the uplink of wireless cellular networks. To do so we begin by building some conditions for the feasibility of the resource (power and bandwidth) allocation problem. We first develop a *reference* feasibility condition (FC), for which the users QoS can be evaluated only by long simulations. To cope with this problem, we propose a sufficient feasibility condition (SFC) which induces a moderate loss of capacity compared to FC. Finally, we establish a rapid and accurate analytical approximation for the users QoS under SFC.

The proposed approach is faster than simulation for streaming traffic since it is based on a *multi-rate Erlang loss model*, whose blocking probabilities can be evaluated by means of the Kaufman-Roberts algorithm (or more simply by Erlang's formula in some particular cases). The proposed method permits also to evaluate analytically the QoS of elastic users (mean throughput and delay) by using a *multi-class processor sharing model*. An interesting question for future work is to evaluate the impact of the shadowing and the mobility of users.

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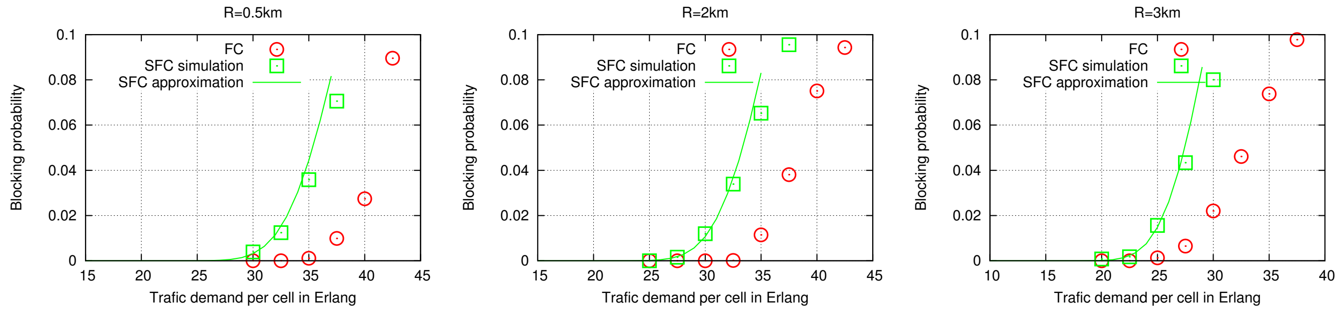

Fig. 1. Comparison of feasibility conditions performance for CDMA uplink for cell radius $R = 0.5, 2, 3$km from top to down respectively.

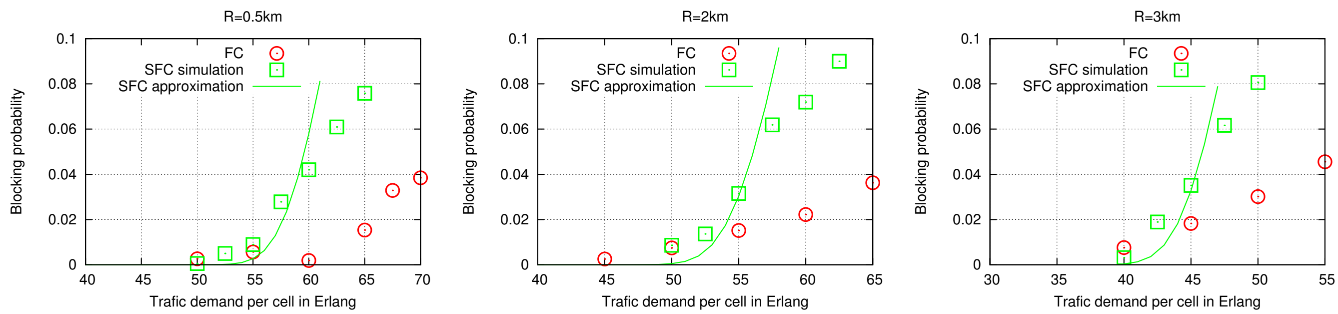

Fig. 2. Comparison of feasibility conditions performance for OFDMA uplink for cell radius $R = 0.5, 2, 3$km from top to down respectively.

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