

Spectral and energy efficiencies of OFDMA wireless cellular networks

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Abstract—We study in the present paper the *spectral efficiency* of OFDMA cellular networks which is a classical indicator of their performance usually calculated by simulations. On the other hand, due to an increasing interest on the environmental impact of these networks, we study also their *energy efficiency* which may be defined as the ratio of the spectral efficiency and the consumed energy.

We give an explicit expression of the signal to interference and noise power ratio (SINR) as function of the user's location. This permits to *calculate easily* the cell spectral efficiency, and thus to *study its variations* with respect to the key parameters of the network such as the cell radius, the propagation characteristics and the single link performance.

We show that the energy efficiency admits a maximum for some value of the transmitted power which is finite and non-null. Moreover, we observe that optimizing the transmitted power permits to *double the energy efficiency*.

Index Terms—Wireless Cellular Networks, Spectral efficiency, Energy efficiency, Performance, Power.

I. INTRODUCTION

The *spectral efficiency* is a key performance indicator of wireless cellular networks. It is used in particular within the 3rd Generation Partnership Project (3GPP) to compare physical layer techniques. On the other hand, there is a recent increasing interest on the environment impact of cellular networks which led to the concept of *energy efficiency* defined as the ratio of the spectral efficiency to the consumed energy. Even though the spectral and energy efficiencies are considered as major indicators, there are no available *analytical methods* for their evaluations. Usually simulations are used, but they make parametric studies aiming to characterize the *sensitivity of these efficiencies* to the network parameters too time-consuming and difficult to analyze.

The objective of the present study is to establish analytical methods permitting to evaluate these efficiencies and to study their sensitivity to the network parameters. We focus on Orthogonal Frequency-Division Multiple Access (OFDMA) systems such as the 3GPP Long Term Evolution (LTE).

A. Related works

The spectral efficiency has originally been defined for real-time services (such as voice) as the ratio of the carried traffic per cell expressed in Erlang to the spectrum bandwidth (see for example [1] and [2]). More recently, cellular networks have to serve elastic bit-rate traffic (such as ftp, web, mail) besides real-time calls. In order to account for such traffic, the carried traffic is replaced by the *throughput* in the definition of the spectral efficiency as in [3]. It is assumed there that the

throughput may be adapted to the signal to noise power ratio (SNR) by choosing the suitable M -QAM modulation scheme.

The *spectral efficiency* may be defined in each location as the ratio of the throughput to the bandwidth of a user assumed alone in the cell and located at the considered location. The *cell spectral efficiency* is then defined as the average of the spectral efficiency with respect to a location uniformly distributed in the cell.

Alouini and Goldsmith [4] introduce the term *area spectral efficiency* to designate the ratio of the cell spectral efficiency to the cell surface. They study its variation with respect to the frequency reuse factor (which equals 1 when all the frequency spectrum is available to all base stations). The sensitivity of the spectral efficiency to other parameters such as the propagation parameters (distance exponent), the cell radius, the transmitted power, is not studied.

The 3GPP publishes reports (e.g., [5]) giving the numerical values of the cell spectral efficiency for different configurations of cellular systems. These values are calculated by time-consuming simulations. Even though simulations give a useful insight on the system performance in particular situations, they are inappropriate for exhaustive parametric studies aiming to investigate the sensitivity of the model to its key parameters. Analytic studies are suitable for such investigations, and they thus constitute a crucial complement to simulations.

Mogensen et al. [6] show that the performance of a single link of LTE may be approximated accurately by a suitable modification of the Shannon formula for AWGN channels. Then they show that the spectral efficiency may be calculated by integrating the link performance formula with respect to the distribution of the signal to interference and noise ratio (SINR). They propose to take the SINR distribution from simulations, but in doing so the spectral efficiency has to be calculated by simulation. We will propose an explicit expression of the SINR as function of the user location which permits to calculate analytically the spectral efficiency.

In order to compare different micro base stations deployment strategies, Richter et al. [7, Eq. (2)] define the *area power consumption* as the consumed power per surface unit (by analogy to the area spectral efficiency of [4]). Observing that the area power consumption goes to infinity when the inter-base station distance goes to 0 or infinity, they deduce that the area power consumption admits a minimum for some non-null and finite value of the inter-base station distance. Unfortunately, the power is calculated to assure only a *coverage* condition, so the interference effect is not taken into account. We shall define the *energy efficiency* which accounts for both coverage

and interference.

B. Paper organization

The remaining part of this paper is organized as follows. The link performance is characterized in Section II. We define the spectral efficiency and establish an analytical method for its calculation in Section III. We introduce the energy efficiency and show that it admits a maximum for some value of the transmitted power in Section IV. Finally, the numerical results are given in Section V.

II. LINK PERFORMANCE

In the present section we consider a single link between a transmitting base station and a receiving user. We assume that each user performs *single user detection*. We assume that the user's bit-rate r is related to his bandwidth w and his signal to noise power ratio SNR by

$$r = w \times \psi(\text{SNR}) \quad (1)$$

for some non-decreasing function ψ . We now show typical examples where the above form of the link performance arises naturally with a specific expression of the function ψ in each case.

A. AWGN and its modifications

A typical example of the function ψ is

$$\psi(\text{SNR}) = b \log_2 \left(1 + \frac{1}{a} \text{SNR} \right) \quad (2)$$

where the constants a and b permit to account for the loss in practical systems compared to the ideal AWGN case (for which the above formula applies with $a = b = 1$). A value $b < 1$ may account for the loss of bandwidth due to signalling [6] and we may take $a \neq 1$ to account for practical coding schemes as shown in [8].

B. Flat fading

Consider a flat fading channel with single input and single output (SISO) antennas. Assume that the fading states are i.i.d and known by the receiver. It is shown in [9, Equation (2)] that the *ergodic capacity* of such a channel is given by (1) where

$$\psi(\text{SNR}) = E \left[\log_2 \left(1 + |S|^2 \text{SNR} \right) \right]$$

where the expectation is taken with respect to the fading state S . In the particular case of Rayleigh fading, $|S|^2$ has exponential distribution, then

$$\begin{aligned} \psi(\text{SNR}) &= \int_0^\infty \log_2(1 + t\text{SNR}) e^{-t} dt \\ &= \frac{1}{\ln 2} e^{1/\text{SNR}} E_1(1/\text{SNR}) \end{aligned}$$

where $E_1(s) = \int_1^\infty \frac{e^{-st}}{t} dt$ is the *exponential-integral function*. In this case, the following approximation is proposed in [10]

$$\psi(\text{SNR}) \approx \log_2 \left(1 + \frac{1}{2} \text{SNR} \right)$$

which has the same form as (2) with $a = 2$.

C. Interference

We shall assume that interference acts as Gaussian noise with equivalent power equal to the sum of the interfering base stations powers (averaged over the fading process). This assumption is pessimistic if we assume that the average power transmitted by the interfering base stations are given. Indeed, using [11, Theorem 18] we may show that, with respect to capacity, the worst noise process distribution (not necessarily white nor Gaussian) with given second moment is the AWGN.

Thus we shall apply formula (1) with SNR replaced by the signal to interference and noise power ratio SINR.

III. SPECTRAL EFFICIENCY

The SINR may be viewed as a function of the location $x \in \mathbb{R}^2$ of the user, which we denote by $\text{SINR}(x)$.

Definition 1: The *spectral efficiency* at a location $x \in \mathbb{R}^2$ is defined as the ratio of the bit-rate to the bandwidth of a user located at x , that is

$$s(x) = \psi(\text{SINR}(x)) \quad (3)$$

where ψ is the function describing the link performance (1).

The spectral efficiency for a subset $A \subset \mathbb{R}^2$ of the network is defined as the average of the spectral efficiency for a random location X uniformly distributed over A , that is

$$S(A) := E[s(X)] = \frac{1}{|A|} \int_A s(x) dx$$

where E designates the mathematical expectation with respect to the random location X and $|A|$ is the surface of A . The spectral efficiency is expressed in bits/s/Hz.

In particular, if A is the cell of a given base station u , then $S(A)$ is called *cell spectral efficiency* and denoted by $S(u)$.

Remark 2: Note that the spectral efficiency may be expressed in bits/s/Hz. Since the Hertz is the inverse of the second, it is correct to express the spectral efficiency in bits. Nevertheless we prefer to express it in bits/s/Hz to insist on the fact that it is a bit-rate per spectrum unit.

A. Calculus

In order to calculate the spectral efficiency, it is crucial to know how does the SINR depends on the location x . The expression of the function $\text{SINR}(x)$ depends not only on the network at hand but also on the considered model. In particular, it is not always easy to get an explicit expression of this function.

1) Model and notation: We describe now a model of an OFDMA cellular network which is simple enough to get a closed form expression of $\text{SINR}(x)$ that however captures some key features of such networks.

Assume that all the base stations (BS) use the same frequency spectrum of bandwidth W and emit the same power \tilde{P} which accounts for the antenna gains and losses. The common channels (not dedicated to a specific user) power equal $\epsilon \tilde{P}$ for some given $\epsilon \in [0, 1]$. Assume moreover that each BS transmits a constant power spectral density, i.e. it allocates to each user a power p proportional to its bandwidth w ; that is

$$p = \frac{w}{W} (1 - \epsilon) \tilde{P} \quad (4)$$

We denote by N be the noise power, that is $N = W N_0$ where N_0 is the noise power spectral density.

Let $L_u(x)$ be the propagation loss between a location x and a BS u (not including the fading effect which is taken into account in the link performance formula (1)). We assume that each BS u serves users in some exclusive geometric cell associated to it which *does not evolve in time*. We will use the same index u to designate the BS and the associated cell. Since all the BS emit the same power, the ratio of the interference to the signal power received at a given location x equals

$$f(x) = \sum_{v \neq u} \frac{L_u(x)}{L_v(x)}, \quad x \in u.$$

Finally, let $L(x)$ be the propagation loss between location x and its serving BS, that is

$$L(x) = L_u(x) \quad \text{when } x \in u.$$

2) *SINR expression*: Each BS u allocates some number of sub-carriers of the total width w from the total spectrum of width W to each user in its cell, in such a way that two different users of the same BS have disjoint subsets of sub-carriers. BS u allocates to each user a power p proportional to its bandwidth w as in (4). Since each BS transmits a constant power spectral density, a user at location $x \in u$ receives interference from each base station $v \neq u$ of power $\frac{w}{W} \tilde{P}/L_v(x)$. We assume that this interference acts as Gaussian noise (see Section II-C), thus

$$\begin{aligned} \text{SINR}(x) &= \frac{p/L_u(x)}{\frac{w}{W}N + \frac{w}{W} \sum_{v \neq u} \tilde{P}/L_v(x)} \\ &= \frac{\frac{w}{W}(1-\epsilon) \tilde{P}/L_u(x)}{\frac{w}{W}N + \frac{w}{W} \sum_{v \neq u} \tilde{P}/L_v(x)} = \frac{1}{\hat{f}(x)} \end{aligned} \quad (5)$$

where for the second equality we use (4) and where for the third equality we introduce the *modified interference factor* $\hat{f}(x)$ defined as follows

$$\hat{f}(x) = \frac{1}{1-\epsilon} \left(\frac{NL(x)}{\tilde{P}} + f(x) \right), \quad x \in u. \quad (6)$$

B. Explicit bounds

In the present section, we assume that the BS are placed on a regular *infinite hexagonal* grid on \mathbb{R}^2 and that the propagation-loss between a BS u and a location x equals

$$L_u(x) = (K|x-u|)^\eta \quad (7)$$

where $\eta > 2$ and $K > 0$ are two given constants.

Proposition 3: The SINR at location x is bounded by

$$\frac{\alpha_1}{L(x)} \leq \text{SINR}(x) \leq \frac{\alpha_2}{L(x)}$$

where

$$\alpha_1 = \frac{1-\epsilon}{N/\tilde{P} + (2/\sqrt{3})^\eta \zeta(\eta-1) \sup g_1} \quad (8)$$

$$\alpha_2 = \frac{1-\epsilon}{N/\tilde{P} + \zeta(\eta-1) \inf g_1} \quad (9)$$

where g_1 is the sum of the inverses of the propagation losses from the six closest interfering BS and ζ is the *Riemann zeta function* given by $\zeta(t) = \sum_{k=1}^{\infty} k^{-t}$, for $t > 1$.

Proof: The interference to signal ratio $f(x)$ is bounded by [12, Prop. 2]

$$L(x) \zeta(\eta-1) \inf g_1 \leq f(x) \leq L(x) \left(2/\sqrt{3}\right)^\eta \zeta(\eta-1) \sup g_1 \quad (10)$$

Then the modified interference factor (6) is bounded by

$$\frac{L(x)}{\alpha_2} \leq \hat{f}(x) \leq \frac{L(x)}{\alpha_1}$$

Equation (5) finishes the proof. \blacksquare

Since the spectral efficiency is an increasing function of the SINR, the above proposition leads to bounds for the spectral efficiency.

IV. ENERGY EFFICIENCY

Let p be the power transmitted by the base station not including the antenna gains and losses. We assume that the base station consumes a power P , which is related to its transmitted power p by the following simple linear relation [7, Eq. (2)]

$$P = cp + d \quad (11)$$

for some positive constants c and d . The *energy efficiency* is defined by

$$\mathcal{E}(p) = \frac{WS(u)}{P} = \frac{WS(u)}{cp + d} \quad (12)$$

where $S(u)$ is the cell spectral efficiency and W is the system bandwidth. The energy efficiency is expressed in bits/s/W. The above equation shows that it may be easily deduced from the cell spectral efficiency, which may be calculated by using the formulae given in the previous sections. Moreover, the following proposition gives the limits of the energy efficiency as the transmitted power goes either to 0 or infinity.

Proposition 4: The energy efficiency tends to 0 when the transmitted power p tends either to 0 or infinity; that is

$$\lim_{p \rightarrow 0} \mathcal{E}(p) = \lim_{p \rightarrow \infty} \mathcal{E}(p) = 0$$

Proof: Let \tilde{P} be the power transmitted by the base station including the antenna gains and losses; that is

$$\tilde{P} = pG$$

where G accounts for the antenna gains and losses. Using (6), we deduce that for any location x ,

$$\lim_{p \rightarrow 0} \hat{f}(x) = \infty, \quad \lim_{p \rightarrow \infty} \hat{f}(x) = \frac{f(x)}{1-\epsilon}$$

then using (5) we get

$$\lim_{p \rightarrow 0} \text{SINR}(x) = 0, \quad \lim_{p \rightarrow \infty} \text{SINR}(x) = \frac{1-\epsilon}{f(x)}$$

Using (3) and (2), we deduce the limits of the spectral efficiency at location x ; that is

$$\lim_{p \rightarrow 0} s(x) = 0, \quad \lim_{p \rightarrow \infty} s(x) = b \log_2 \left(1 + \frac{1-\epsilon}{af(x)} \right)$$

On the other hand, it is easy to see that $s(x)$ is increasing with p , thus using the monotone convergence theorem we deduce the limit of the the cell spectral efficiency

$$\lim_{p \rightarrow 0} S(u) = 0, \quad \lim_{p \rightarrow \infty} S(u) = \frac{b}{|u|} \int_u \log_2 \left(1 + \frac{1-\epsilon}{af(x)} \right) dx$$

Equation (12) then implies that $\lim_{p \rightarrow 0} \mathcal{E}(p) = 0$. On the other hand, using (10) we may easily show that the integral in the right-hand side of the above equation is finite. Equation (12) then implies that $\lim_{p \rightarrow \infty} \mathcal{E}(p) = 0$ which finishes the proof. \blacksquare

We deduce from the above proposition the following corollary.

Corollary 5: The energy efficiency admits a maximum for some non-null and finite value p^* of the transmitted power.

V. NUMERICAL RESULTS

A. Model specifications

We assume that the BS are placed on a regular *infinite hexagonal* grid on \mathbb{R}^2 . Let Δ be the distance between two adjacent BS, then the surface area of a given cell (hexagon; i.e., subset of the plane whose points are closer to a given BS than to any other) equals $\sqrt{3}\Delta^2/2$. We may approximate the hexagonal cell with the (virtual) disc whose area is equal to that of the hexagon. The radius of this disc is then equal to

$$R = \Delta \sqrt{\frac{\sqrt{3}}{2\pi}}$$

which we call *cell radius*.

We consider a propagation loss function in the form (7) where $\eta = 3.38$ and $K = 8667\text{km}^{-1}$ which corresponds to the Hata model [13] for an *urban* environment with BS height 50m and user height 1.5m. Each BS transmits a power $P = 43\text{dBm}$ ($= 20\text{W}$) and has an antenna with gain 12dBi, thus the transmitted power equals $\tilde{P} = 55\text{dBm}$ when we account for the antenna gain. A portion $\epsilon = 0.12$ of this power is used by common channels (not dedicated to a specific user). The noise power at mobile equals $N = -103\text{dBm}$. The cell radius equals $R = 250\text{m}$. The link performance has the form (2) where $b = 0.5$ and $a = 2$. The consumed power P is related to the transmitted power p by the linear relation (11) where $c = 21.45$ and $d = 354.44\text{W}$ [7, §III.A].

We make some parametric studies where we vary some of the above parameters while the other ones equal their default values.

B. Results

The cell spectral efficiency and the energy efficiency equal respectively

$$S = 0.93\text{bits/s/Hz}, \quad \mathcal{E} = 6 \times 10^3\text{bits/s/W}$$

1) *Propagation exponent*: Fig. 1 represents the cell spectral efficiency S as function of the propagation exponent denoted η (see (7)) as well as the bounds deduced from Proposition 3. We observe that S increases with η until some $\eta_0 \in [4, 4.5]$ then decreases. In order to interpret this result, recall first that the spectral efficiency is an increasing function of $1/\hat{f}(x)$ where the modified interference factor $\hat{f}(x)$ is given by (6). Note that $\hat{f}(x)$ is the sum of two terms. Firstly, the *noise term*

$$\frac{N}{(1-\epsilon)\tilde{P}}L(x) = \frac{N}{(1-\epsilon)\tilde{P}}(K|x-u|)^\eta, \quad x \in u \quad (13)$$

increases with η when $K|x-u| > 1$ (which is the case in the major part of the cell). Secondly, the *interference term*

$$\frac{f(x)}{(1-\epsilon)} = \sum_{v \neq u} \frac{L_u(x)}{L_v(x)} = \sum_{v \neq u} \left(\frac{|x-u|}{|x-v|} \right)^\eta, \quad x \in u \quad (14)$$

decreases with η . When η is small, the interference term is preponderant and thus $\hat{f}(x)$ decreases with η . On the other hand, when η is large, the noise term is preponderant and therefore $\hat{f}(x)$ increases with η which is coherent with our numerical result in Fig. 1.

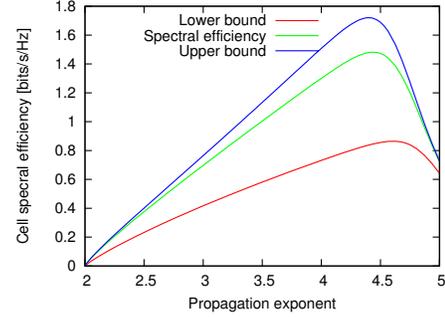


Fig. 1. Cell spectral efficiency as function of the propagation exponent.

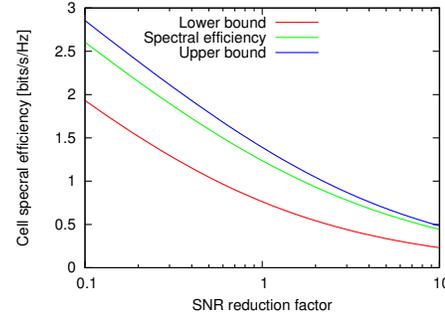


Fig. 2. Cell spectral efficiency as function of the SNR reduction factor.

2) *SNR reduction factor*: Fig. 2 represents the cell spectral efficiency as function of the SNR reduction factor denoted a in the link performance formula (2). A technology evolution may be characterized in some cases by a decrease of the SNR reduction factor a . Fig. 2 shows that dividing a by 10 induces a gain in the spectral efficiency by a factor of about 2 or 3. This is due to the logarithmic dependence of the performance (2) on the SNR.

3) *Cell radius*: Fig. 3 represents the cell spectral efficiency S as function of the cell radius R . We observe that there are two regimes. For small R (up to about 2 km) S is independent of the cell radius; whereas for larger R , S decreases rapidly with R . In order to interpret this result, we use again the fact that the spectral efficiency is an increasing function of $1/\bar{f}(x)$ where the modified interference factor $\bar{f}(x)$ is given by (6). Taking the average of this equation over the cell we get

$$\bar{\hat{f}} = \frac{1}{1-\epsilon} \left(\frac{N\bar{L}}{\tilde{P}} + \bar{f} \right) \quad (15)$$

where we denote by $\bar{\hat{f}}$, \bar{L} and \bar{f} the average of $\hat{f}(x)$, $L(x)$ and $f(x)$ over the cell. The right-hand side of the above equation is the sum of two terms which we call *noise* and *interference terms* respectively.

When the cell radius R is small, the interference term is preponderant in the sum in (15) and thus

$$\bar{\hat{f}} \simeq \frac{1}{1-\epsilon} \bar{f}$$

which is independent of the cell radius. This theoretical result is coherent with the numerical observation that the cell spectral efficiency is constant in this case. On the other hand, when the cell radius R is large, the noise term is preponderant in

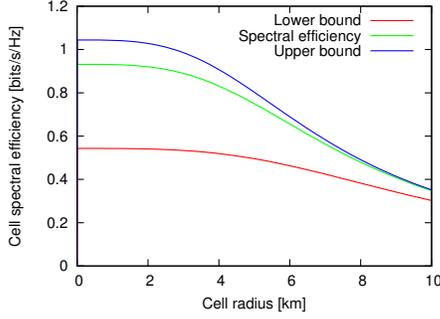


Fig. 3. Cell spectral efficiency as function of the cell radius.

the sum in (15) and thus

$$\bar{f} \simeq \frac{N}{(1-\epsilon)\bar{P}} \bar{L} = \frac{N}{(1+\eta/2)(1-\epsilon)\bar{P}} (KR)^\eta$$

which increases with R . This result is coherent with the numerical observation that the cell spectral efficiency decreases rapidly with R in this case. In the above analysis we recognize the two well known regimes called *interference limited* and *noise limited* regimes.

4) *Bounds*: In Figures 1, 2 and 3 we represent also the bounds of the cell spectral efficiency deduced from Proposition 3. Note first that these bounds capture the essential effects of the variation of the considered parameters on the spectral efficiency. Moreover note that the upper bound gives a good estimate of the exact value, since it is at most 10% away from it. Alouini and Goldsmith [4] give also lower and upper bounds, but they are unfortunately far from the exact value when the frequency reuse factor is low. In particular, for a frequency reuse factor equal to 1, their lower bound is about the third of the simulated value and their upper bound is about two times the exact value. We see that our bounds are much closer to the exact value than theirs.

5) *Energy efficiency*: Fig. 4 represents the energy efficiency as function of the transmitted power. As expected from Corollary 5, we observe that the energy efficiency admits a maximum for some non-null and finite value of the transmitted power. The value of this *optimal power* depends on the configuration of the network. In particular, the numerical value of the optimal power \bar{P}^* (including the antenna gain and loss) is about 30, 35 and 40dBm for the cell radii $R = 0.250$, 0.5 and 1km respectively. Fig. 4 shows that this optimal power permits approximately to double the energy efficiency compared to the current situation where $\bar{P} = 55$ dBm and $\mathcal{E} = 6 \times 10^3$ bits/s/W.

VI. CONCLUSION

We establish an analytical method to calculate the spectral and energy efficiencies of OFDMA cellular networks. We use this method to analyze the *sensitivity of these efficiencies* to the network parameters. We observe that the spectral efficiency firstly increases with respect to the propagation exponent (i.e. the exponent of the distance in the propagation-loss) and then decreases when the propagation exponent exceeds some value which we calculate numerically. Regarding the cell radius, the cell spectral efficiency remains constant until some value of the cell radius and then decreases. These two regimes correspond

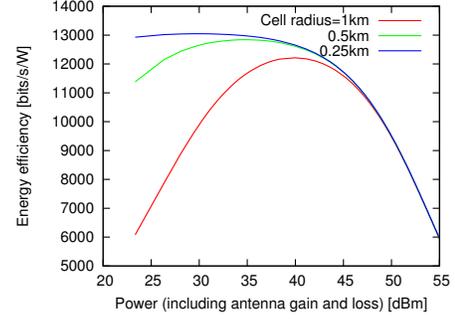


Fig. 4. Energy efficiency as function of the transmitted power.

to the so-called interference and noise limited regimes. We also give an upper and a lower bound for the spectral efficiency. The upper bound gives a good estimate of the exact value, since it is at most 10% away from it. Finally, we show that the energy efficiency admits an optimum for some value of the transmitted power and may be doubled if we choose this power suitably.

ACKNOWLEDGMENT

The author thanks Hassene Ben Hammouda at CEA as well as Man-Fai Wong and Azeddine Gati at Orange Labs for useful discussions.

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