Wireless Netw (2011) 17:247–262
DOI 10.1007/s11276-010-0277-8

User's mobility effect on the performance of wireless cellular networks serving elastic traffic

Mohamed Kadhem Karray

Published online: 14 September 2010
© Springer Science+Business Media, LLC 2010

Abstract The objective of the present paper is to give an analytic approximation of the performance of elastic traffic in wireless cellular networks accounting for user's mobility. To do so we build a Markovian model for users arrivals, departures and mobility in such networks; which we call WET model. We firstly consider intracell mobility where each user is confined to remain within its serving cell. Then we consider the complete mobility where users may either move within each cell or make a handover (i.e. change to another cell). We propose to approximate the WET model by a Whittle one for which the performance is expressed analytically. We validate the approximation by simulating an OFDMA cellular network. We observe that the Whittle approximation underestimates the throughput per user of the WET model. Thus it may be used for a conservative dimensioning of the cellular networks. Moreover, when the traffic demand and the user speed are moderate, the Whittle approximation is good and thus leads to a precise dimensioning.

Keywords Communication system performance · Radiocommunication · Markov processes · Mobility · Speed

1 Introduction

The performance evaluation of wireless cellular networks may be decomposed into three subproblems.

M. K. Karray (✉)
France Telecom, Research and Development Division,
38/40 rue du Général Leclerc, 92794 Issy-Moulineaux, France
e-mail: mohamed.karray@orange-ftgroup.com

First, we have to characterize the performance of a single radio link. In doing so we have to take into account the radio signal variations due to multi-path fading. This may be carried by using the tools of *information theory*.

Once the single link performance is characterized, we should take into account the interference between the different links which depends on the relative geographic positions of the users. In particular, this interference should be taken into account when *resource* (power and bandwidth) is *allocated* to the users. In order to optimize the network performance, the resource allocation may be adapted to the channel conditions, for example by allocating the resource to the user with the best radio conditions. Such allocation is said *opportunistic*.

Finally, the dynamics induced by the users arrivals, mobility and departures should be taken into account in order to calculate the throughput or delay perceived by the users in the long run of the network. This may be done by using tools from *queueing theory*.

A careful separation of the time scales between the above three subproblems is required to get a global performance evaluation method. The mobility of the users in wireless cellular networks have effects on each of the above three subproblems (or time-scales).

The effect of user's mobility at the information theory and resource allocation time scales has already been investigated, at least qualitatively, as will be discussed in Sect. 6.

The objective of the present paper is to study the effect of mobility of the users at the queueing theory time-scale. More precisely, we assume that the single level performance (given by information theory) doesn't depend on user's speed. Moreover, we assume a given resource allocation scheme which may be either opportunistic or not, but doesn't depend on mobility. The network carries elastic traffic (non-real-time data),[1] which accepts fluctuations of the bit-rates.

Even though the above assumptions seem to be too simplistic, they permit to study and understand the effect of user's speed at the queueing theory time-scale. In other words, we decompose the complex problem at hand into subproblems as it is usual in science.

The remaining part of this paper is organized as follows. We begin by discussing the related literature in Sect. 1.1. Then we describe the basic model and tools in Sect. 2. In Sect. 3 we study a model where mobility is only allowed within each cell. In the two extreme mobility regimes corresponding to null and infinite user's speed, we give analytical expressions of the performance. But for intermediate user's speeds, no analytical results are known, so we propose in Sect. 4 approximate expressions for the performance by using an approximate model. Finally, we validate the proposed approximation in Sect. 5 by simulating an Orthogonal Frequency-Division Multiple Access (OFDMA) cellular network serving elastic traffic users which are subject to mobility. In Appendix 1 we describe a particular model for mobility called *completely aimless mobility*.

1.1 Related work

There are many publications on performance evaluation of wireless cellular networks serving elastic traffic, see for example [2–4, 12, 14, 22, 26, 28].

Nevertheless only few authors study the effect of the *mobility* of users on performance, which is the main concern of the present paper.

Bonald et al. [10] studies the effect of some *local* variations of the user feasible rates induced typically by fading. Even though, these variations may be viewed as related to some local mobility, they don't account for the (large scale) *geographic mobility* of users.

For elastic traffic, the mean number of users may grow unboundedly in the long run of the system. This situation has to be avoided; in which case we say that the system is *stable*.

Bonald and Proutière [13, Sect. 2] proposes a *Whittle model* to evaluate the impact of mobility of elastic users on performance of cellular networks. Roughly speaking, in this model each user has some volume of data to transmit at each visited location, and he does not move from this location until the end of the transmission of the required volume (for a precise definition of the Whittle model see for example [27]). Unfortunately, in this model the mobility of the users is influenced by a congestion in the network. In particular, when the network approaches instability, user mobility is being frozen.

A more suitable model, where mobility is independent from users services, is studied in [16, 17], whose results are summarized in [11, Sect. 4]. Considering a stationary and ergodic mobility process, the stability conditions as well as stochastic comparisons of performance are given for α-fair scheduling schemes. It is shown in particular that mobility increases capacity. (It is also shown that capacity decreases with the fairness index α, but this is not in the scope of the present paper.)

A similar model, called *WET model*, is considered in [7]. In the WET model each user has some given volume of data to transmit during the whole sojourn in the system independently from his mobility.

Unfortunately neither of the previous four references give an explicit expression of the performance, except in the two extreme cases of motionless and infinitely-rapid users. In the case of non-null and finite mean user speed, the performance of the WET model may only be deduced from simulations. Analytical expressions are needed for the network *dimensioning*; i.e., evaluating the minimal number of base stations assuring some quality of service (typically a given throughput per user) for some given traffic demand per surface unit.

1.2 Our contribution

In the present work, we approximate analytically the performance of the WET model in the case of non-null and finite mean user speed. The idea is to make an *appropriate modification of the WET model* into a *Whittle* one, for which analytical results are available.

We validate the approximation by simulating an OFDMA cellular network. We first consider intracell mobility. We observe that the throughput per user given by the Whittle approximation is smaller than that of the WET model. Moreover, for a moderate traffic demand (small compared to the critical value corresponding to instability), the approximation is very good.

Then we consider the complete mobility where users may either move within each cell or make a handover (i.e. change to another cell). We observe that the complete mobility performs better than intracell mobility. On the other hand, when the mean user speed is moderate, the performance for complete mobility may be approximated by that of intracell mobility.

Since the Whittle approximation underestimates the throughput per user of the WET model, it may be used for a conservative dimensioning of the cellular networks. Moreover, when the traffic demand and the user speed are moderate, the Whittle approximation is good and thus leads to a precise dimensioning.

[1] Streaming services (i.e. real-time such as voice calls, video streaming, etc.) are not considered in the present study.

2 Basic model and tools

In this section we develop mathematical tools for the spatio-temporal analysis of the wireless networks with elastic traffic. In particular we introduce and study a *Wireless Elastic Traffic* (WET) model.

2.1 System state and its evolution

We will consider a wireless network composed of a finite set of base stations (BSs). We assume that each BS u serves users is some exclusive geometric *cell* associated to it, which is a bounded subset of the plane $\mathbb{R}^2$, and which does not evolve in time. With a slight abuse of notation, we will use the same letter for the BS and its cell. In particular, $x \in u$ means that the location x is served by BS u.

Let $\mathbb{D}$ be the union of all the cells. Elements $x \in \mathbb{D}$ denote *geographic locations of users* in the system. Configurations $\{x_i\} \subset \mathbb{D}$ of users in the system are identified with the corresponding counting measures $v = \sum_i \varepsilon_{x_i}$; where ε_x is the Dirac measure defined by $\varepsilon_x(A) = 1$ if $x \in A$ and 0 otherwise. Consequently $v(A)$ is the number of users in the set $A \subset \mathbb{D}$. In particular, $v(u)$ is the number of users in the cell u. We denote by $\mathbb{M}$ the set of *all finite configurations of users* in the system (i.e., finite counting measures on $\mathbb{D}$).

We will describe the temporal evolution of the configuration of users in $\mathbb{D}$ by a pure jump Markov process, which takes values in $\mathbb{M}$. This process evolves because of users arriving, moving or leaving the system, with only one such event being possible at a time.

2.2 Wireless elastic traffic (WET) model

2.2.1 *Arrivals and departures*

We will model the process of call arrivals to and departures from $\mathbb{D}$ as a birth-and-death process: for a given subset $A \subset \mathbb{D}$, interarrival times to A are independent of everything, exponential random variables with mean $1/\lambda(A)$, where $\lambda(\cdot)$ is some given intensity measure of arrivals to $\mathbb{D}$ per unit of time. In homogeneous traffic conditions, we can take $\lambda(dx) = \lambda dx$, where λ is the mean number of arrivals per unit of area and per unit of time.

We assume that each arrival brings to the system some volume of data (amount of bits that has to be sent or received), which is modeled by an independent of everything else, exponential random variable with parameter μ. Users are served by the BS according to some transmission rate assignment policy. They depart immediately after having transmitted their volumes. The departures times depend on the number and positions of the users and the rate assignment policy.

The measure

$$\rho(dx) = \frac{\lambda(dx)}{\mu}$$

is called *traffic demand density* (expressed in kbps[2] per surface unit).

Remark 1 The assumption that the interarrival time (i.e. the duration between two *new arrivals* to the system) has an exponential distribution permits to make the model analytically tractable. The assumption that the volume brought by each arrival has an exponential distribution may be in some cases relaxed due to the so-called insensitivity property [5, p. 123], but this is not in the scope of the present paper.

2.2.2 *Mobility*

We will model the mobility of users is $\mathbb{D}$ by some Markov process. Specifically, we assume that users move independently of each other in $\mathbb{D}$. The sojourn duration of a given user at location $x \in \mathbb{D}$ is exponentially distributed with parameter $\lambda'(x)$. Any user finishing its sojourn at location x, is routed to a new location dy according to some probability kernel $p'(x, dy)$, where $p'(x, \mathbb{D}) = 1$.

The above description corresponds to a Markov process on $\mathbb{D}$ with the following generator (of the individual user mobility): $\lambda(x, dy) = \lambda'(x)p'(x, dy)$ called *mobility kernel*. We will always assume that this Markov process is ergodic and will denote its invariant distribution by $\sigma(\cdot)$; it satisfies:

$$\begin{aligned} &\sigma(\mathbb{D}) = 1 \\ &\int_A \lambda(x, \mathbb{D})\sigma(dx) = \int_{\mathbb{D}} \lambda(x, A)\sigma(dx), \quad A \subset \mathbb{D} \end{aligned} \tag{1}$$

where the first equation assures that σ is a probability measure and the second one is the so-called *balance equation*.

We give in Appendix 1 the explicit expressions of the parameters $\lambda'(x)$ and $p'(x, dy)$ for a particular example of mobility model: the so-called *completely aimless mobility*. Note that in this particular case $\lambda(x, dy)$ is proportional to the *average speed of users* denoted υ. We will always assume such proportionality. It follows in particular that the solution σ of (1) is independent of υ.

2.2.3 *Transmission rates*

When the configuration of the users in the network is $v \in \mathbb{M}$, each user located at some $x \in \mathbb{D}$ is allocated a *transmission rate* which we will denote by $r_x(v)$. In cellular networks, the expression of the function $r_x(v)$ derives from

[2] The abbreviation kbps designates "Kilo-bit per second".

some condition of the feasibility of the resource (power and bandwidth) allocation problem as we shall see in Sect. 5.1.1. In this context, a sufficiently general form of the transmission rates is as follows

$$r_x(v) = \frac{1}{h(v(u(x)))\gamma(x)}, \quad x \in \mathbb{D}, v \in \mathbb{M} \tag{2}$$

where $h : \mathbb{N} \to \mathbb{R}_+$ and $\gamma : \mathbb{D} \to \mathbb{R}_+$ are two given functions and $u(x)$ is the cell containing x. The function γ is related to the geometry of interference, whereas the function h depends on the type of resource allocation; for example whether it is opportunistic or not. (We shall see in Sect. 5.1.1 that in the particular case of a non-opportunistic allocation, we have $h(n) = n$.)

We introduce for future reference the function $\mathcal{H}$(s) defined for $s > 0$ by

$$\mathcal{H}(s) = \frac{E[H(X+1)]}{E[H(X)]}, \quad H(M) = \begin{cases} \prod_{k=1}^{M} h(k) & \text{if } M \geq 1 \\ 1 & \text{if } M = 0 \end{cases} \tag{3}$$

where X is a Poisson random variable with parameter s. In the particular case $h(n) = n$, we have

$$\mathcal{H}(s) = \frac{1}{1-s} \tag{4}$$

(for a proof see [24, p. 99]).

2.2.4 Generator

Assume that the system is in some state $v \in \mathbb{M}$. An arrival of a user at position y brings the system to state $v + \varepsilon_y$, and this occurs with rate $\lambda(dy)$. A departure of a user from position x brings the system to state $v - \varepsilon_x$, and this occurs with rate $\mu\ r_x(v)v(dx)$. Finally a mobility of a user from position x to position y brings the system to state $v - \varepsilon_x + \varepsilon_y$, and this occurs with rate $\lambda(x, dy)v(dx)$. Thus the generator q of the *Wireless Elastic Traffic* (WET) process is: for $v \in \mathbb{M}$, $\Gamma \subset \mathbb{M}$

$$\begin{aligned} q(v,\Gamma) = &\int_{\mathbb{D}} 1(v + \varepsilon_y \in \Gamma)\lambda(dy) \\ &+ \int_{\mathbb{D}} 1(v - \varepsilon_x \in \Gamma)\mu r_x(v)v(dx) \\ &+ \int_{\mathbb{D}\times\mathbb{D}} 1(v - \varepsilon_x + \varepsilon_y \in \Gamma)\lambda(x, dy)v(dx) \end{aligned} \tag{5}$$

where the three integrals in the right-hand side of the above equation correspond to arrivals, departures and mobility of users respectively. We shall always assume that the generator q is regular (for definition and for sufficient conditions for this assumption to hold see for example [24, Sect. 6.2.2]).

Our WET model for wireless networks serving elastic traffic is different form the Whittle model proposed for such networks in [13, Sect. 2] (see Sect. 1.1 for more discussion). If the WET model is more realistic, its inconvenience is that we cannot evaluate its performance explicitly. For this reason, following the idea of [10], we shall consider the following two extreme cases: *no mobility* and *infinite mobility*. In the former case the WET model is equivalent to a spatial version of the Whittle model of [13, Sect. 2], while in the latter case we proceed by a *separation of the time scales* of mobility and arrivals/departures.

2.2.5 Performance indicators

If the process describing the evolution in time of the users configuration isn't ergodic, then the mean number of users grows unboundedly in the long run of the system. This *instable* situation has to be avoided in real-life networks.

One distinguish two milestones of the analytical *evaluation of the network performance*: identification of its stability region, and the evaluation of the steady state characteristics (e.g. the mean number of users, the mean throughput and delay perceived by each user).

2.3 Whittle approximation

In the case of non-null and finite mobility, the performance of the WET model may only be deduced from simulations. We aim to approximate analytically this performance. The idea is to make a modification of the WET model into a *Whittle* one, for which analytical results are available. More specifically, we consider the following modified version of the WET generator (5)

$$\begin{aligned} q'(v,\Gamma) = &\int_{\mathbb{D}} 1(v+\varepsilon_y \in \Gamma)\lambda(dy) \\ &+ \int_{\mathbb{D}} 1(v-\varepsilon_x \in \Gamma)\mu r_x(v)v(dx) \\ &+ \int_{\mathbb{D}\times\mathbb{D}} 1(v-\varepsilon_x+\varepsilon_y \in \Gamma)\lambda(x,dy)h(v(u(x)))^{-1}v(dx) \end{aligned} \tag{6}$$

where we have modified only the mobility component by introducing 'artificially' $h(v(u(x)))^{-1}$, just to make the process of *Whittle* type [24, Sect. 7.1.1] ; and thus analytically tractable as we will see later. We shall validate this approximation by comparing its results to those of the simulation of the original WET model.

2.4 Intracell and complete mobility

When the users move in a cellular network, they may some times change their serving base station, a phenomenon called *handover*. We will distinguish such *intercell mobility* from the mobility inside the cell, which we call *intracell mobility*.

More specifically we distinguish the following variants of the WET model. In the *intracell mobility model* the users move inside each cell but never cross the frontier between the cells. Concretely, when a user arrives to the edge of a cell, he may only backtrack. An example of such model is described in Appendix 1.2. Of course, this is not a very realistic model, but it will help us to understand the key phenomena and relations for the *complete mobility model* which comprises both intracell and intercell mobility.

3 Performance for intracell mobility

We begin our analysis of the performance of the WET model by considering only intracell mobility. By assumption each user remains in the same cell during all its call. Since moreover the service rates (2) in a given cell depend only on its users, we may study the evolution of the configuration of users in each cell individually, i.e. independently from that in the other cells.[3]

Along the present section, we are interested in the process describing the evolution of the configuration of the users in a given cell u. Note that in order to simplify the notation, we shall omit to specify explicitly the dependence of some parameters (such as the traffic demand in the cell) on u.

3.1 No mobility case

Assume now that the users don't move; i.e., each user is served at his arrival location.

Proposition 1 *Consider a particular cell u and let $\rho = \lambda(u)/\mu$ be the corresponding traffic demand. Let*

$$\bar{\gamma} = \frac{1}{\lambda(u)} \int_u \gamma(x)\lambda(dx), \quad \rho_c = \frac{1}{\bar{\gamma}}.$$

If

$$\rho < \rho_c \lim_{n\to\infty} \frac{n}{h(n)} \tag{7}$$

then the corresponding WET model without mobility is stable. In this case the expected number of users in cell u, the mean delay and throughput per user are given respectively by

$$\bar{N} = \frac{\rho}{\rho_c}\mathcal{H}\left(\frac{\rho}{\rho_c}\right), \quad \bar{T} = \frac{\bar{N}}{\lambda(u)}, \quad \bar{r} = \frac{1}{\mu\bar{T}}.$$

In the particular case $h(n) = n$, the above formulae become

$$\bar{N} = \frac{\rho}{\rho_c - \rho}, \quad \bar{T} = \frac{1}{\mu(\rho_c - \rho)}, \quad \bar{r} = \rho_c - \rho. \tag{8}$$

Proof Note that the WET model without mobility is a spatial birth-and-death model equivalent to a spatial processor sharing queue. Let $\rho'(\cdot)$ be the measure on $\mathbb{D}$ defined by

$$\rho'(A) = \frac{1}{\mu}\int_A \gamma(x)\lambda(dx), \quad A \subset \mathbb{D}.$$

We deduce from [20] and [15, Proposition 3.1] that if

$$\rho'(u) < \lim_{n\to\infty} \frac{n}{h(n)}$$

then the corresponding WET model without mobility is stable. In this case, it is shown in [24, Sect. 7.2.4] that in steady state the expected number of users in a subset A of a cell u and the *mean delay* and *throughput* per user are given respectively by

$$\bar{N}(A) = \rho'(A)\mathcal{H}(\rho'(u)), \quad \bar{T}(A) = \frac{\bar{N}(A)}{\lambda(A)}, \quad \bar{r}(A) = \frac{1}{\mu\bar{T}(A)}$$

where the function $\mathcal{H}$ is given by (3). (Note that the second equality is due to Little's theorem [6].) Observing that

$$\rho'(u) = \frac{1}{\mu}\int_u \gamma(x)\lambda(dx) = \rho\bar{\gamma} = \frac{\rho}{\rho_c}$$

finishes the proof of the first part of the proposition. For the particular case $h(n) = n$, it is enough to recall that the function $\mathcal{H}$ is given by (4). □

The right-hand side of Eq. (7) is called *critical traffic demand*.

3.2 Infinite mobility

Suppose that the mobility of users is so fast that we can reasonably assume that during the periods of time when the number of users is constant, each user receives a service rate that is averaged over its mobility. More precisely, we assume that all the users in cell u receive the same service rate equal to

$$\int_u r_x(v)\sigma(dx) = \frac{\int_u \frac{1}{\gamma(x)}\sigma(dx)}{h(v(u))}$$

[3] Nevertheless each cell is not isolated from the rest of the network, since the effect of the interference from the other cells is indirectly taken into account in (2) as we shall see in Sect. 5.1.1.

where σ is the stationary distribution of the mobility kernel [i.e. σ is solution of (1)]. Then cell u may be considered as a processor sharing queue with arrival rate $\lambda(u)$, mean data volume per user $1/\mu$ and service rate given by the above equation.

Proposition 2 *Consider a particular cell u and the corresponding WET model with infinite mobility. The results of Proposition 1 hold true with ρ_c replaced by*

$$\rho_{c_\infty} = \overline{\gamma^{-1}} = \int_u \frac{1}{\gamma(x)} \sigma(dx). \tag{9}$$

In particular, the stability condition is

$$\rho < \rho_{c_\infty} \lim_{n \to \infty} \frac{n}{h(n)}. \tag{10}$$

Proof Note that the WET model with infinite mobility is equivalent to a processor sharing queue. Let ρ'_∞ be defined by

$$\rho'_\infty = \frac{\lambda(u)}{\mu \int_u \frac{1}{\gamma(x)} \sigma(dx)}.$$

We deduce from [20] and [15, Proposition 3.1] that if

$$\rho'_\infty < \lim_{n \to \infty} \frac{n}{h(n)}$$

then the corresponding WET model with infinite mobility is stable. The end of the proof is then similar to that of Proposition 1; in particular note that

$$\rho'_\infty = \frac{\lambda(u)}{\mu \int_u \frac{1}{\gamma(x)} \sigma(dx)} = \frac{\rho}{\overline{\gamma^{-1}}} = \frac{\rho}{\rho_{c_\infty}}.$$

□

3.3 Comparison

We aim now to compare the performance of the models with null and infinite mobility.

Proposition 3 *Assume that the arrival intensity $\lambda(\cdot)$ is proportional to $\sigma(\cdot)$, i.e.; $\sigma(\cdot) = \lambda(\cdot)/\lambda(\mathbb{D})$. Then*

$$\rho_c \leq \rho_{c_\infty}.$$

Let $\bar{r}$ and $\bar{r}_\infty$ be the mean throughput per user for the models with null and infinite mobility respectively. Then, for a given traffic demand ρ,

$$\bar{r} \leq \bar{r}_\infty.$$

Proof The first inequality follows from the Jensen's inequality. For the second inequality, consider the particular case $h(n) = n$. We deduce from (8) that

$$\bar{r} = \rho_c - \rho \leq \rho_{c_\infty} - \rho = \bar{r}_\infty.$$

For the case of general $h(.)$ see [7, Corollary A.5]. □

We deduce from the above proposition that the mean throughput per user for the model with infinite mobility is larger than for that with no mobility. We shall observe numerically the more general result that the throughput per user increases with the mean user speed υ.

3.4 Intermediate speed

Unfortunately, in the case of intermediate speeds $(0 < \upsilon < \infty)$, there is no known analytical results about the performance of the WET model. We shall study a useful approximation in the next section. The results of the simulations reported in Sect. 5.3.1 below lead to the following observations about the WET model:

1. The stability condition of the WET model for a mean user speed $\upsilon > 0$ seems to be identical to that for infinite mobility, i.e. (10).
2. For a given traffic demand, the throughput per user of the WET model increases with the mean user speed υ.
3. For a moderate traffic demand, the throughput per user of the Whittle approximation (6) agrees well with that of the WET model (5).

The first and second results are proved mathematically in [11, Sect. 4.2.2] and [10, Sect. VI], respectively. The third observation is new.

4 Whittle approximation performance

We aim now to derive the performance of the Whittle model described in Sect. 2.3. The results in the present section apply for either intracell or complete mobility models. Indeed, we will show in Sect. 4.2 that these two mobility models are equivalent within the Whittle framework.

4.1 Performance

We assume that the following system of equations

$$\lambda(\mathbb{D}) = \int_{\mathbb{D}} \frac{\mu}{\gamma(x)} \rho'(dx)$$

$$\int_A \lambda(x, \mathbb{D}) \rho'(dx) + \int_A \frac{\mu}{\gamma(x)} \rho'(dx) = \int_{\mathbb{D}} \lambda(x, A) \rho'(dx) + \lambda(A) \tag{11}$$

(for all $A \subset \mathbb{D}$) admits a unique solution $\rho'(\cdot)$ such that $\rho'(\mathbb{D}) < \infty$. We call the measure $\rho'(\cdot)$ the *modified traffic density*.

Remark 2 We now show that Equations (11) are the balance equations for some extension of the mobility kernel $\lambda(x, dy)$. To do so we introduce a " virtual" state $o \notin \mathbb{D}$ which can be seen as a location outside the space $\mathbb{D}$, and which represents the location of calls arriving to or leaving the system. Denote $\overline{\mathbb{D}} = \mathbb{D} \cup \{o\}$. We extend the mobility kernel $\lambda(x, dy)$ on $\overline{\mathbb{D}}$ as follows

$$\lambda(o, A) = \lambda(A), \quad \lambda(x, \{o\}) = \frac{\mu}{\gamma(x)}, \quad x \in \mathbb{D}, A \subset \mathbb{D}.$$

The extended kernel is called *traffic kernel*. Then (11) is just a reformulation of the corresponding balance equations; i.e.

$$\rho'(o) = 1, \quad \int_A \lambda(x, \overline{\mathbb{D}})\rho'(dx) = \int_{\overline{\mathbb{D}}} \lambda(x, A)\rho'(dx), \quad A \subset \overline{\mathbb{D}}.$$

Proposition 4 *Consider the Whittle Markov process with generator (6). If for all cell u*

$$\rho'(u) < \lim_{n\to\infty} \frac{n}{h(n)} \tag{12}$$

then the Whittle process is stable. In this case, the expected number of users in cell u, the mean delay and throughput per user are given respectively by

$$\bar{N} = \rho'(u)\mathcal{H}(\rho'(u)), \quad \bar{T} = \frac{\bar{N}}{\lambda(u)}, \quad \bar{r} = \frac{1}{\mu\bar{T}}.$$

In the particular case $h(n) = n$, the above formulae become

$$\bar{N} = \frac{\rho'(u)}{1 - \rho'(u)}, \quad \bar{T} = \frac{\rho'(u)}{\lambda(u)(1 - \rho'(u))}, \quad \bar{r} = \frac{\rho(u)}{\rho'(u)} - \rho(u).$$

Proof For the expected number of users see [24, Proposition 41]. The expression of the mean delay follows from Little's formula. Finally, the mean throughput equals the mean volume divided by the mean delay. □

It is interesting to reformulate the stability condition in the above proposition in a form analogous to (7), i.e. traffic demand per cell less than a critical value. This is done in the following corollary.

Corollary 1 *Consider the Whittle Markov process with generator (6). For each cell u, Condition (12) may be written as the traffic demand in cell u, denoted ρ, satisfying*

$$\rho < \rho_{c_\upsilon} \lim_{n\to\infty} \frac{n}{h(n)}$$

where

$$\rho_{c_\upsilon} = \frac{1}{\rho'_1(u)}$$

and $\rho'_1(dx)$ is solution of Equations (11) with $\lambda(dx)$ replaced by $\lambda_1(dx) = \lambda(dx)\,/\rho(u)$. In case of stability the expected number of users in cell u, the mean delay and throughput per user are given respectively by

$$\bar{N} = \frac{\rho}{\rho_{c_\upsilon}}\mathcal{H}\left(\frac{\rho}{\rho_{c_\upsilon}}\right), \quad \bar{T} = \frac{\bar{N}}{\lambda(u)}, \quad \bar{r} = \frac{1}{\mu\bar{T}}.$$

In the particular case $h(n) = n$, the above formulae become

$$\bar{N} = \frac{\rho}{\rho_{c_\upsilon} - \rho}, \quad \bar{T} = \frac{1}{\mu(\rho_{c_\upsilon} - \rho)}, \quad \bar{r} = \rho_{c_\upsilon} - \rho.$$

Proof Consider the Whittle model and a particular BS u. It is easy to see from (11) that when the arrival intensity $\lambda(dx)$ is multiplied by a constant a, then the modified traffic demand density $\rho'(dx)$ is multiplied by the same constant. Taking $a = 1/\rho(u)$ shows that

$$\rho'(dx) = \rho(u)\rho'_1(dx)$$

where $\rho'_1(dx)$ is solution of (11) with $\lambda(dx)$ replaced by $\lambda_1(dx) = \lambda(dx)/\rho(u)$. Denoting $\rho_{c_\upsilon} = \frac{1}{\rho'_1(u)}$ and using Proposition 4 finishes the proof. □

4.1.1 Retrieving the null mobility result

It is easy to see that the WET and Whittle models are identical when the mean user speed υ is null. Let's check that the results of Corollary 1 with $\upsilon = 0$ are coherent with those of Proposition 1. To do so, it is enough to check that

$$\rho_{c_0} = \rho_c.$$

Proof It is easy to see that, when $\upsilon = 0$,

$$\rho'_1(dx) = \frac{\gamma(x)}{\mu\rho(u)}\lambda(dx).$$

Thus

$$\rho'_1(u) = \frac{1}{\mu\rho(u)}\int_u \gamma(x)\lambda(dx) = \frac{1}{\lambda(u)}\int_u \gamma(x)\lambda(dx) = \bar{\gamma}.$$

Therefore

$$\rho_{c_0} = \frac{1}{\rho'_1(u)} = \frac{1}{\bar{\gamma}} = \rho_c.$$

□

4.1.2 Retrieving the infinite mobility result

We aim now to study the limit of the Whittle performance when the mean user speed goes to infinity. To this end, proceeding as in [24, Proof of Proposition 33], we deduce that when $\upsilon \to \infty$ the modified traffic demand density goes to

$$\rho'(dx) = \frac{\lambda(u)}{\int_u \frac{\mu}{\gamma(y)}\sigma(dy)}\sigma(dx).$$

In particular, when $\lambda(dx)$ is replaced by $\lambda_1(dx) = \lambda(dx)$ / $\rho(u)$, we get

$$\rho'_1(dx) = \frac{\sigma(dx)}{\int_u \frac{1}{\gamma(y)}\sigma(dy)}.$$

Thus

$$\rho_{c_\infty} = \frac{1}{\rho'_1(u)} = \int_u \frac{1}{\gamma(x)}\sigma(dx)$$

which agrees with (9).

4.2 Complete versus intracell mobility

We shall show that the invariant distributions of the Whittle model has a *product form* from which it will follow that the complete and the intracell mobility models are equivalent within the Whittle framework.

Proposition 5 *Assume that the stability condition* (12) *holds true for all cell u. Then the Whittle generator* (6) *admits as invariant measure the measure on* $\mathbb{M}$ *with density*

$$\Psi(v) = \prod_u \left(\prod_{i=1}^{v(u)} h(i)\right)$$

with respect to the distribution of the Poisson process on $\mathbb{D}$ *with mean measure* ρ'.

Proof This follows from [24, Proposition 28]. □

Let ρ' and ρ'_u be the solutions of Equations (11) for complete mobility and intracell-u mobility models respectively. Moreover, let π and π_u be the invariant distributions of the Whittle generator (6) for complete mobility and intracell-u mobility models respectively.

Assume that, for all cell u in the network, ρ'_u is the restriction of ρ' to u (which is the case for the completely aimless mobility as assured by Proposition 9). In this case, Proposition 5 shows that

$$\pi = \prod_u \pi_u.$$

It follows that the expectations of the number of users within cell u with respect to π and π_u are the same. The performance (*mean delay* and *throughput* per user) for the complete and the intracell mobility models are then identical within the Whittle framework. But is such result true within the WET framework?

4.2.1 Back to the WET framework

Since analytical results are not yet available for the WET model, the comparison of complete and intracell mobility within the WET framework can only be carried by simulations. The results of the simulations reported in Sect. 5.3 below lead to the following observations about the WET model:

1. When the mean user speed is moderate, the performance of complete mobility may be approximated by that of intracell mobility.
2. When the mean user speed is high, the performance of complete mobility is better than that of intracell mobility.

5 Elastic traffic in OFDMA

In this section we will show how the ideas developed in the previous sections can be used to analyze the performance of an OFDMA cellular network serving elastic traffic users, which are subject to mobility. The main goal is to evaluate the impact of their average (mobility) speed on throughput, when taking into account the geometry of interference in the whole network.

5.1 Feasible configurations of users

First we briefly recall the problem of resource allocation in wireless cellular networks.

5.1.1 Feasible configurations

It is natural to identify the feasible configurations of bit-rates in the network $\mathbb{D}$ by studying the feasibility of resource (power and bandwidth) allocation problem. In this approach, a given configuration of bit-rates is *feasible* if there exists an allocation of powers and bandwidths to users which respects the information theory constraint as well as the maximal power and total bandwidth constraints. However, solving this resource allocation problem for a large network is a very complicated task.

Sufficient condition It is shown in [8] that a sufficient condition for the feasibility of resource allocation is that each base station u respects the following inequality for the users of its own cell

$$\sum_{x \in v \cap u} \gamma(x) r_x \leq 1 \tag{13}$$

where r_x is the bit-rate for user x, and $\gamma(\cdot)$ is some non-negative function of the user location. (The above condition may be derived by assuming that all base stations emit with their maximal powers and the users have equal power spectral density; see [8, Sect. IV.B]. Note also that the effect of the interference from the other cells is taken into account in (13) since this condition assures the feasibility of the global resource allocation.)

The above approach suggests the following choice of the bit-rate allocation

$$r_x = \frac{1}{\nu(u)\gamma(x)}, \quad x \in u$$

where $\nu(u)$ is the number of users in cell u. Note that the above allocation is a particular case of the general form (2) with the function h there given by $h(n) = n$. The above allocation is called *non-opportunistic* by opposition to *opportunistic* allocations which *allocate the resource to the user with the most favorable fading state* (see [9]). The general form (2) permits to account for such allocations, but we shall only consider non-opportunistic allocations in the simulations reported in the present paper.

Cell discretization In order to get a discrete model, we partition each cell into several rings around the base station. Then we take as representative value in each ring of the function $\gamma(\cdot)$ in (13) the average of $\gamma(x)$ where x is distributed according to $\lambda(dx)$. In the most simple case, when u is not partitioned at all, we get a unique value, denoted $\bar{\gamma}$, which equals

$$\bar{\gamma} = \frac{1}{\lambda(u)} \int_u \gamma(x)\lambda(dx).$$

In this case the geometric model boils down to a discrete one.

5.2 Model specification

5.2.1 *Network architecture*

We consider the radio part of the downlink in wireless cellular OFDMA networks. In order to obtain numerical values, we consider the most popular hexagonal model, where the base stations (BS) are placed on a regular hexagonal grid. Let R be the radius of the disc whose area is equal to that of the hexagonal cell served by each base station, and call R the *cell radius*. We take R = 0.525 km.

In order to avoid the border effects we consider the network that is " wrapped around"; i.e., deployed on a torus comprising 4 × 4 = 16 cells. Each cell is decomposed into three (equally thick) rings around the base station.

We assume a propagation loss $L(r) = (Kr)^{\eta}$, with $\eta = 3.38$ and K = 8,667 where r is the distance between the transmitter and the receiver.

The system bandwidth equals W = 5 MHz.

Base stations are equipped with omnidirectional antennas having a gain 9 dBi and no loss. The BS maximal total power equals 43 dBm; thus $\tilde{P} = 43 + 9 = 52$ dBm when we account for antenna gain and loss. The common channel power $\hat{P}$ is the fraction $\varepsilon = 0.12$ of $\tilde{P}$ and the ambient noise power equals $N = -103$ dBm.

5.2.2 *Call arrivals and data volumes*

We consider elastic traffic calls arriving spatially uniformly over the cell; i.e. $\lambda(dx) = \lambda\, dx$. Each arrival brings to the system some volume of data of mean $1/\mu$ which we will specify in the next section.

5.2.3 *User mobility*

We assume the *completely aimless* mobility model described in Appendix 1. It yields the uniform stationary distribution σ of user location in the network.

Note that if we replace the generator q given in Equation (5) by q/μ then the corresponding invariant distribution remains unchanged. Thus all we have to specify in the simulations are the traffic demand per cell $\rho = \lambda(u)/\mu$ expressed in Mbit/s and the ratio υ/μ expressed in Mbit × km/s. Besides the null speed case $\upsilon = 0$, we make simulations with

$$\upsilon/\mu = 10^{-3}, 10^{-2}, \ldots, 10^{2}.$$

If the mean data volume $1/\mu = 1$ Mbit, then the above values correspond respectively to *mean user speeds*

$$\upsilon = 3.6, 36, \ldots, 3600000\text{km/h}$$

The high values of the user speed permit to approach numerically the theoretical infinite speed case. Note that if the mean data volume $1/\mu = 1$ Gbit, then the mean user speeds are respectively

$$\upsilon = 0.0036, 0.036, \ldots, 3600\text{km/h}.$$

5.2.4 *Dynamic simulations*

The simulations are performed using Matlab. Due to the discretization of each cell into three rings, the *configuration of users* in the system is simply the number of users in each ring of each cell.

We start from an empty system (no users) and make 10^6 transitions (arrivals, departures or mobility of users) of the Markov process describing the evolution of the configuration of users in the system. The number of transitions is sufficiently large in order to attain the stationary regime. The expectation of the configuration of users under the stationary distribution is then estimated from the simulated trajectory by using the ergodic theorem [18, Theorem 6.2, p. 363]. Applying Little's formula [6] we get the mean delay per user. Finally, the mean throughput per user is calculated as the ratio of the data volume average and the delay.

5.3 Numerical results

5.3.1 *Intracell mobility*

We consider first the intracell mobility model.

Figure 1 shows the mean throughput per user for the WET model as function of the traffic demand per cell for $\upsilon/\mu = 10^{-3}, 10^{-2}, \ldots, 10^{2}$. Note first that the numerical result for $\upsilon/\mu = 100$ agrees with the analytical result for infinite mobility. Note moreover that for a given traffic demand, the throughput per user increases with speed, thus performance is improved by mobility in accordance with the theoretical result of [10, Sect. VI]. On the other hand, Fig. 1 (down) shows that the stability condition for the three values $\upsilon/\mu = 1, 10, 100$ is the same as that for the analytical model with infinite mobility in accordance with the theoretical result of [11, Sect. 4.2.2].

A possible intuition justifying this result is as follows. When the WET model approaches instability, the call duration (delay) of each user increases whereas its motion is not altered (by opposition to what happens in the Whittle model where user mobility freezes when instability is approached; for more discussion see Sect. 1.1). At instability, the call duration is infinite and the user traverses all the network (many times) exactly as if its speed were infinite.

Figure 2 shows the results obtained by *simulations* for the WET model and those obtained analytically for the Whittle *approximation*. We observe that the throughput given by the Whittle approximation is a lower bound of that of the WET model. Moreover, for a small traffic demand (small compared to the critical value corresponding to instability), the performance of the Whittle approximation agrees well with that of the WET model.

In particular, when the *traffic demand goes to zero*, the two models have the *same performance*, i.e. give the same mean throughput per user. The throughput corresponding to a null traffic demand is a key performance parameter

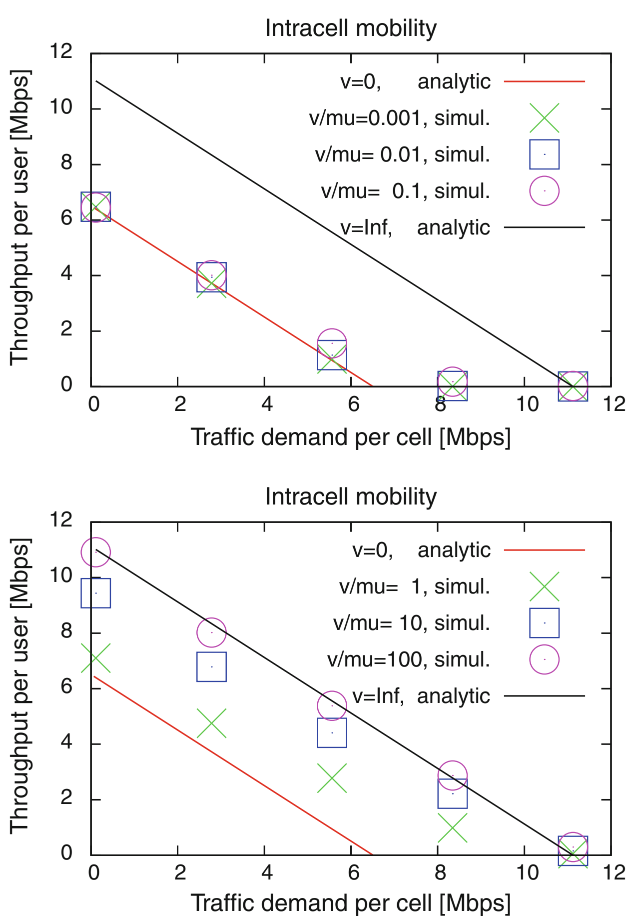

Fig. 1 WET model with intracell mobility

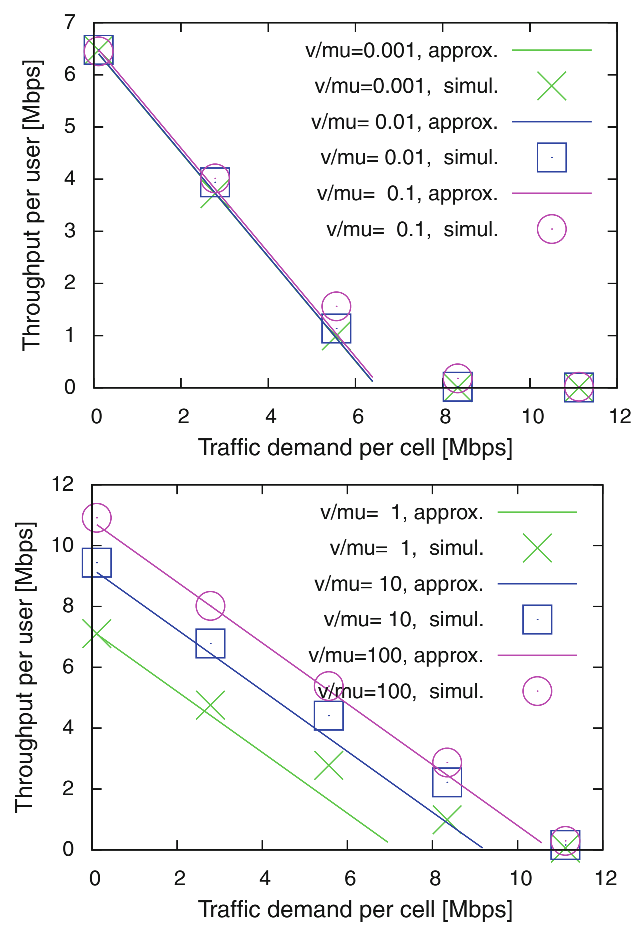

Fig. 2 Whittle approximation versus WET simulations for intracell mobility

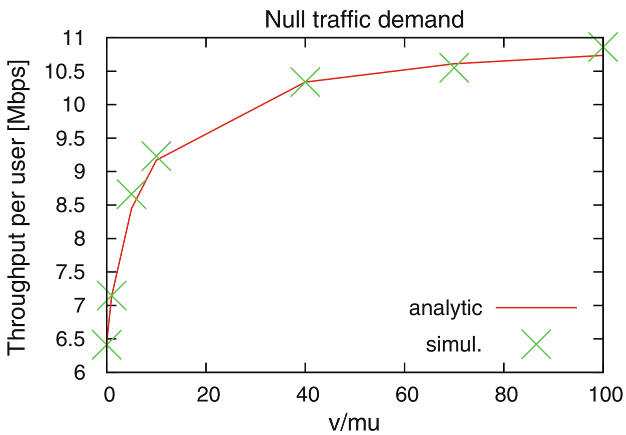

Fig. 3 Whittle approximation versus WET simulations for null traffic demand

since it gives also the slope of the throughput as function of the traffic demand as we see in Fig. 2. We represent this throughput (corresponding to a null traffic demand) as function of the speed obtained by *simulations* for the WET model and analytically for the Whittle *approximation* in Fig. 3. This confirms our previous observation that the two models agree in this case and shows how does the throughput corresponding to a null traffic demand increase with speed.

5.3.2 *Complete mobility*

Figure 4 shows the results of the simulations of the WET model with complete and intracell mobility. We see globally that the complete mobility gives a larger (or equal) throughput than the intracell mobility. This result complements naturally the previous observation that the throughput increases with the user's speed when only intracell mobility is considered. In fact, the performance improves when mobility within each cell increases, and *improves even more* when mobility between the cells is allowed. Thus considering only intracell mobility gives a conservative estimate of performance.

We observe on Fig. 4(top) that the improvement of complete mobility compared to intracell mobility is negligible when $v/\mu < 0.1$. Thus when the mean user *speed is low enough*, the performance of complete mobility may be *well approximated* by that of intracell mobility.

6 Other time scales

Till now we considered only the queueing theory time scale. We consider now the other two time scales: information theory and resource allocation.

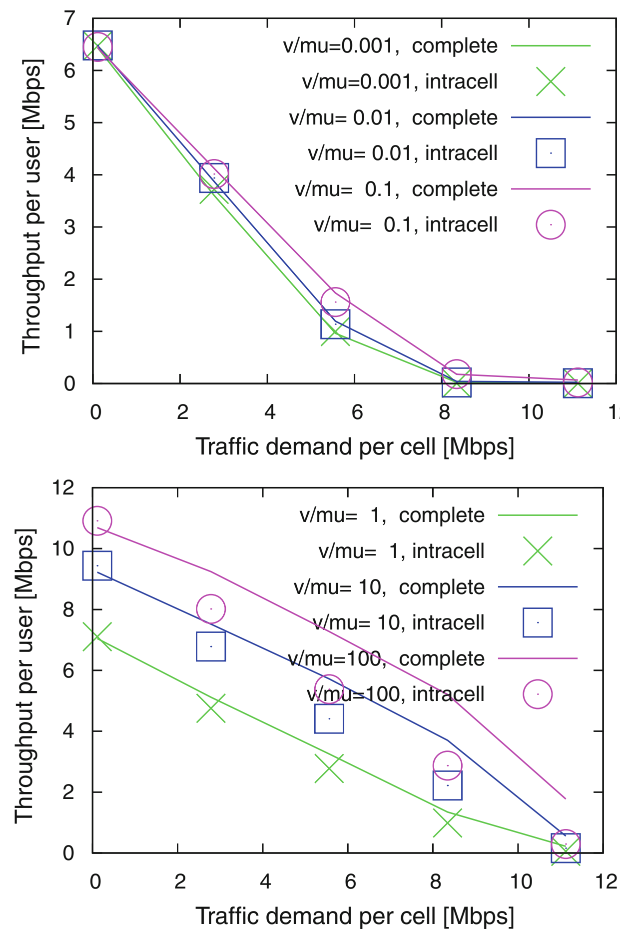

Fig. 4 WET model for complete and intracell mobility

6.1 Single link (Information theory)

The wireless channel with multi-path fading may be modelled as a linear time-variant system characterized by a transfer function $T(f, t)$ which depends on frequency f and time t. A crucial characteristic of such system, called *coherence time*, is the typical time interval for the transfer function $T(f, t)$ to change significantly, for a fixed frequency f. It may be shown that the coherence time is roughly inversely proportional to user's speed (see for example [21], Chap. 9). This formalizes the intuitions that the more the user moves, the more the channel is variable.

On the other hand, the wireless channel may be viewed as a channel with different states (typically the values of the fading at successive time instants). In such channels, from the information theory point of view, the knowledge of the channel state at least by the receiver (called Channel State Information at Receiver CSIR) is crucial to counteract the harmful effect of its variability (see for example [1, Fig. 7]).

The increase of the channel variability makes it more difficult to track its state, and thus to loose the improvement due to CSIR. Thus the user's mobility is harmful from this point of view.

6.2 Resource allocation

A resource allocation is said *opportunistic* when at each time, each bandwidth portion is allocated to the user with the most favorable fading state. This requires the knowledge of the channel state at the transmitter (called Channel State Information at Transmitter CSIT).

It is observed in [25, Sect. 6.2], that the opportunistic allocation scheme is suitable for slow varying fading as in the case of *fixed (or pedestrian)* users for example. For fast varying fading, due to *high mobility* of users, it is difficult to track the rapidly varying channel, and thus only a non-opportunistic allocation scheme is possible. On the other hand, the opportunistic allocation improves performance (compared to non-opportunistic allocation) as shows for example in [9]. Thus, we deduce from the above analysis that mobility is harmful for performance when we consider its effect on the possibility of an opportunistic resource allocation scheme.

6.3 Perspective

It is interesting in future work to couple the fact that mobility at the queueing theory time-scale ameliorates the performance with the fact that mobility degrades performance at the information theory and resource allocation time-scales. In other words, what is the cumulative effect of user's mobility on the global performance?

7 Conclusion

We give an analytic approximation of the performance of elastic traffic in wireless cellular networks accounting for the mean user speed. To do so we consider a Markovian model for such networks called WET model.

We firstly consider intracell mobility where users are confined to remain within their serving cell. The performances of this model for the extreme mobility regimes of null and infinite user speeds are expressed analytically. Unfortunately, no analytical results are known for intermediate user speed. Thus, we propose for such speeds to approximate the WET model by a Whittle one for which the performance is known analytically. The numerical results show that the throughput per user of the WET model is larger than that of the Whittle model. Moreover, for a moderate traffic demand (small compared to the critical value corresponding to instability), the throughput per user of the WET model may be well approximated by that of the Whittle model.

Then we consider the complete mobility where users may either move within each cell or make a handover (i.e. change to another cell). The simulation of the WET model in this context lead to the following observations. When the mean user speed is moderate, the performance of complete mobility may be approximated by that of intracell mobility. On the other hand, when the mean user speed is high, the performance of complete mobility is better than that of intracell mobility.

We deduce that the Whittle approximation may be used for a conservative dimensioning of cellular networks (since it underestimates the user's throughput). Moreover, when the traffic demand and the user speed are moderate, the agreement between the Whittle approximation and the WET model with complete mobility is good. In this context, the Whittle approximation leads to a precise dimensioning.

Acknowledgments The author thanks Prof. Bartłomiej Błaszczyszyn at INRIA (Institut National de Recherche en Informatique et Automatique) for motivating discussions and help.

Appendix 1: Completely aimless mobility

We present in this appendix 1 mobility model based on the following assumptions (see [29]):

- The speeds of the users are considered as *random vectors* in $\mathbb{R}^2$ and are assumed independent and identically distributed.
- The speed *direction* of a typical user is a random variable which is uniformly distributed in $[0,2\pi]$.

Following the authors of [23] we call this model *completely aimless* mobility.

Let V be the speed *magnitude* of a typical user, F be its cumulative distribution function and $\upsilon = E[V]$ be its *mean*.

1.1 Sojourn duration

We are interested in the user's sojourn duration in a given geographic zone of area A and perimeter L. Much as in [29], we derive a relation between the average sojourn duration (which will be useful in the construction the mobility kernel in the following section) and the average speed.

We are interested in the users crossing an infinitesimal element dl of the border (for example from outside to inside) within an infinitesimal duration dt. Such users are located in a rectangle of sides dl and $V\cos\alpha dt$, as illustrated in Fig. 5, where:

- V is the user's speed magnitude;
- and α is the angle formed by the user's speed vector and the perpendicular to dl.

Integrating over V and α, we obtain the average number of users crossing an element dl of the border of the zone, from outside to inside, during dt

$$\int_{-\pi/2}^{\pi/2}\int_{0}^{+\infty} VF(dV)\cos\alpha\frac{d\alpha}{2\pi}\rho dldt = \frac{\upsilon\rho}{\pi}dldt$$

where ρ is the density of users per surface unit, F is the cumulative distribution function of the user's speed magnitude and $\upsilon = E[V]$. Then the average number of users crossing the zone border per time-unit denoted λ (which is the average arrival rate of users to the zone) is given by

$$\lambda = \upsilon\rho L/\pi \tag{14}$$

where L is the perimeter of the zone.

Denote τ the average sojourn duration of a user in a zone and $\bar{M}$ the average number of users in the zone. By Little's formula, we have

$$\bar{M} = \lambda\tau$$

which gives

$$\tau = \bar{M}\lambda^{-1} = \rho A\frac{\pi}{\upsilon\rho L} = \frac{\pi A}{\upsilon L} \tag{15}$$

where A is the surface of the zone. For a disc of radius R, we have $A/L = R/2$.

The authors of [23] consider an exponential distribution for the sojourn duration. This assumption is justified by [19].

1.2 Intracell mobility

The cell is modeled by a disc of radius R which is divided into J rings. Each ring denoted by some $j \in \{1, \ldots, J\}$ is delimited by discs with radii $r_{j\text{-}1}$ and r_j where $r_0 = 0$ and $r_J = R$. Let $A_j = \pi(r_j^2 - r_{j\text{-}1}^2)$ be the surface of ring j. Of course J should be large enough to capture correctly the geometry of the problem.

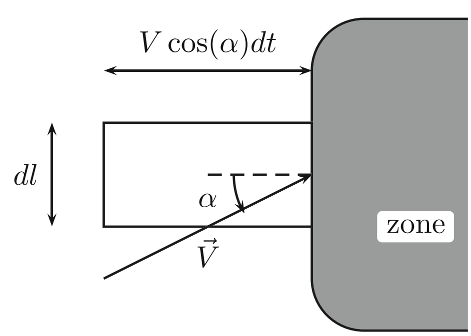

Fig. 5 Rectangle containing customers crossing an element dl of the border during dt

Consider the case where mobility is within a given cell. Denote λ'_j the inverse of the average sojourn duration of users at ring j. Applying (15) gives

$$\lambda'_j = \frac{\upsilon}{\pi}\frac{L_j}{A_j} = 2\upsilon\frac{r_j + r_{j-1}}{A_j}, \quad j = 1, \ldots, J-1$$
$$\lambda'_J = \frac{\upsilon}{\pi}\frac{L_J}{A_J} = 2\upsilon\frac{r_{J-1}}{A_J}$$

A user finishing its sojourn at ring j is routed:

- either to ring $j-1$ or to ring $j+1$ with respective probabilities $p'_{j,j-1} = r_{j\text{-}1}/(r_j + r_{j-1})$ and $p'_{j,j+1} = r_j/(r_j + r_{j-1})$, if $j = 2, \ldots, J-1$;
- to ring 2 with probability 1, if $j = 1$;
- to ring $J-1$ with probability 1, if $j = J$.

Define the *mobility kernel* (λ_{jk}) on $\{1, \ldots, J\}$ by

$$\lambda_{jk} = \lambda'_j p'_{jk}, \quad j,k \in \{1,\ldots,J\}.$$

We deduce from the above results that

$$\begin{aligned}\lambda_{j,j-1} &= 2\upsilon\frac{r_{j-1}}{A_j}, \quad j = 2,\ldots,J\\ \lambda_{j,j+1} &= 2\upsilon\frac{r_j}{A_j}, \quad j = 1,\ldots,J-1.\end{aligned} \tag{16}$$

Proposition 6 *The mobility kernel* $(\lambda_{jk};j,k \in \{1, \ldots, J\})$ *where the* λ_{jk} *are given by* (16) *admits*

$$\sigma_j = \frac{A_j}{\pi R^2}, \quad j = 1,\ldots,J \tag{17}$$

as invariant probability measure, i.e. $(\sigma_j, j \in \{1, \ldots, J\})$ *is solution of the following balance equations*

$$\sigma_j\sum_k \lambda_{j,k} = \sum_k \sigma_k\lambda_{k,j}, \quad j \in \{1,\ldots,J\}. \tag{18}$$

Proof Equation (18) may be written as follows

$$\begin{cases}\sigma_j(\lambda_{j,j-1} + \lambda_{j,j+1}) = \sigma_{j-1}\lambda_{j-1,j} + \sigma_{j+1}\lambda_{j+1,j} & \text{for } j = 2,\ldots,J-1\\ \sigma_1\lambda_{1,2} = \sigma_2\lambda_{2,1}\\ \sigma_J\lambda_{J,J-1} = \sigma_{J-1}\lambda_{J-1,J}\end{cases}$$

For the rates (16) we get

$$\begin{cases}\sigma_j\frac{r_j+r_{j-1}}{A_j} = \sigma_{j-1}\frac{r_{j-1}}{A_{j-1}} + \sigma_{j+1}\frac{r_j}{A_{j+1}} & \text{for } j = 2,\ldots,J-1\\ \sigma_1\frac{r_1}{A_1} = \sigma_2\frac{r_1}{A_2}\\ \sigma_J\frac{r_{J-1}}{A_J} = \sigma_{J-1}\frac{r_{J-1}}{A_{J-1}}\end{cases}$$

which clearly admits σ given by (17) as solution. □

We introduce a " virtual" state 0 which can be seen as a location outside the cell, and which represents the location of calls arriving to or leaving the cell. We consider now some arrival rates denoted λ_{0j} and some departure rates denoted λ_{j0}. We call $(\lambda_{jk};j, k \in \{0, 1, \ldots, J\})$ the *traffic kernel*; which may be seen as an extension of the mobility kernel $(\lambda_{jk};j, k \in \{1, \ldots, J\})$ to $\{0, 1, \ldots, J\}$.

Proposition 7 *Consider the motion rates (16), and let $\lambda_{0j} > 0$ and $\lambda_{j0} > 0$ be the arrival and departure rates respectively. Then for each speed $\upsilon \geq 0$, the following traffic equations*

$$\rho_0 = 1 \text{ and } \rho_j \sum_{k=0}^{J} \lambda_{jk} = \sum_{k=0}^{J} \rho_k \lambda_{kj}, \quad j \in \{1, \ldots, J\} \tag{19}$$

(associated to the traffic kernel ($_{jk}$;j, k ∈ {0, 1, …, J})) admit a unique solution. □

Proof The traffic kernel (λ_{jk};j, k ∈ {0, 1, …, J}) is irreducible by the positivity of the arrival and departure rates. Since the state space {0, 1, …, J} is finite, the Markov process associated to the traffic kernel is positive recurrent and admits an invariant measure ρ with positive terms and unique up to a multiplicative factor (see [18]). Hence (19) admit a unique solution.

1.3 Intercell mobility

Let λ'_u be the inverse of the average sojourn duration of users within cell u. Applying (15) we get

$$\lambda'_u = \frac{\upsilon}{\pi}\frac{L}{A} = \frac{2\upsilon}{\pi R} \tag{20}$$

where the perimeter equals $L = 2\pi R$ and the area equals $A = \pi R^2$. If each base station has six neighbors as in the toric hexagonal model, then a user finishing its sojourn in cell u is routed to a neighboring cell v with probability

$$p'_{u,v} = \frac{1}{6}.$$

Define the mobility kernel $\lambda_{u,v}$ on the set of cells by

$$\begin{aligned} \lambda_{u,v} &= \lambda'_u p'_{u,v} \\ &= \frac{1}{6}\lambda'_u = \frac{\upsilon}{3\pi R} \end{aligned} \tag{21}$$

for each pair of neighboring cells u, v.

1.4 Complete mobility

Consider now a network of hexagonal cells such that each one has exactly six neighbors. Each cell is approximated by a disc and divided into J rings. The cells are indexed by $u \in \mathcal{U} = \{1, \ldots, U\}$, and the rings by $j \in \mathcal{J} = \{1, \ldots, J\}$. The ring j of the cell u is indexed by $uj \in \mathcal{U} \times \mathcal{J}$.

Remark 3 We may alternatively index the rings of cell u by $(u - 1)J + 1, \ldots (u - 1)J + J$. Hence each ring is identified by some location $x = \{1, \ldots, U \times J\}$. From a given such location x, we may retrieve the index of the corresponding cell u and ring j by the Euclidean division

$$x - 1 = (u - 1)J + (j - 1), \quad 1 \leq j \leq J.$$

Denote λ'_{uj} the inverse of the average sojourn duration of users in the ring uj. Applying (15) gives

$$\lambda'_{uj} = \frac{\upsilon}{\pi}\frac{L_j}{A_j} = 2\upsilon\frac{r_j + r_{j-1}}{A_j}, \quad u \in \mathcal{U}, j \in \mathcal{J}.$$

A user finishing its sojourn in ring uj is routed:

- to either ring $u(j - 1)$ or ring $u(j + 1)$ with respective probabilities $p'_{uj,u(j-1)} = r_{j-1}/(r_j + r_{j-1})$ and $p'_{uj,u(j+1)} = r_j/(r_j + r_{j-1})$, if $j = 2, \ldots, J - 1$;
- to ring $u2$ with probability 1, if $j = 1$;
- to either ring $u(J - 1)$ or ring vJ, where v is a neighbor of u, with respective probabilities $p'_{uJ,u(J-1)} = r_{J-1}/(r_J + r_{J-1})$ and $p'_{uJ,vJ} = \frac{1}{6} r_J/(r_J + r_{J-1})$, if $j = J$.

Define the *mobility kernel* ($\lambda_{uj,vk}$) on $\mathcal{U} \times \mathcal{J}$ by

$$\lambda_{uj,vk} = \lambda'_{uj} p'_{uj,vk}, \quad u, v \in \mathcal{U}, j, k \in \mathcal{J}.$$

We deduce from the above results that

$$\begin{cases} \lambda_{uj,u(j-1)} = 2\upsilon\frac{r_{j-1}}{A_j}, & j = 2, \ldots, J \\ \lambda_{uj,u(j+1)} = 2\upsilon\frac{r_j}{A_j}, & j = 1, \ldots, J - 1 \\ \lambda_{uJ,vJ} = \frac{1}{3}\upsilon\frac{r_J}{A_J}, & v \text{ is a neighbor of } u. \end{cases} \tag{22}$$

The result of Proposition 6 may be easily extended to the complete mobility case as follows.

Proposition 8 *The mobility kernel $(\lambda_{uj,vk}; uj, u, v \in \mathcal{U}, j, k \in \mathcal{J})$ given by (22) admits*

$$\sigma_{uj} = \sigma_j = \frac{A_j}{\pi R^2}, \quad u \in \mathcal{U}, j \in \mathcal{J}$$

as invariant probability measure, i.e. $(\sigma_{uj}, u \in \mathcal{U}, j \in \mathcal{J})$ is solution of the following balance equations

$$\sigma_{uj} \sum_{v \in \mathcal{U}, k \in \mathcal{J}} \lambda_{uj,vk} = \sum_{v \in \mathcal{U}, k \in \mathcal{J}} \sigma_{vk}\lambda_{vk,uj}, \quad u \in \mathcal{U}, j \in \mathcal{J}.$$

Proof Besides the proof of Proposition 6, it remains to show that

$$\sigma_J \left[\lambda_{uJ,u(J-1)} + \sum_v \lambda_{uJ,vJ}\right] = \sigma_{J-1}\lambda_{u(J-1),uJ} + \sigma_J \sum_v \lambda_{vJ,uJ}$$

which is equivalent to

$$\sigma_J \lambda_{uJ,u(J-1)} = \sigma_{J-1}\lambda_{u(J-1),uJ}$$

which holds true. □

We introduce a "virtual" location 0 which can be seen as a location outside the cell, and which represents the location of calls arriving to or leaving the network. We consider now some arrival rates denotes $\lambda_{0,uj}$ and departure rates denoted $\lambda_{uj,0}$. We call $(\lambda_{uj,vk}; uj, vk \in (\mathcal{U} \times \mathcal{J}) \cup \{0\})$ the *traffic kernel*; which may be seen as an extension of the mobility kernel $(\lambda_{uj,vk}; uj, vk \in \mathcal{U} \times \mathcal{J})$ to $(\mathcal{U} \times \mathcal{J}) \cup \{0\}$.

The following proposition shows the relation between the invariant measures of the traffic kernels associated to intracell and complete mobility models respectively.

Proposition 9 *Assume that the arrival and departure rates don't depend on the particular cell but only on the ring in which they occur; i.e.* $\lambda_{0,uj} = \lambda_{0,j}$ *and* $\lambda_{uj,0} = \lambda_{j,0}$. *If* $(\rho_j; j \in \mathcal{J} \cup \{0\})$ *is solution of the traffic equations (19), then* $(\rho_{uj}; uj \in (\mathcal{U} \times \mathcal{J}) \cup \{0\})$ *defined by*

$$\rho_{u0} = \rho_0 = 1, \quad \rho_{uj} = \rho_j, \quad u \in \mathcal{U}, j \in \mathcal{J} \tag{23}$$

is solution of the traffic equations associated to the traffic kernel $(\lambda_{uj,uk}; uj, uk \in (\mathcal{U} \times \mathcal{J}) \cup \{0\})$; *i.e.* $\rho_{u0} = 1$ *and for all* $u \in \mathcal{U}, j \in \mathcal{J}$

$$\rho_{uj} \sum_{vk \in (\mathcal{U} \times \mathcal{J}) \cup \{0\}} \lambda_{uj,vk} = \sum_{vk \in (\mathcal{U} \times \mathcal{J}) \cup \{0\}} \rho_{vk} \lambda_{vk,uj}. \tag{24}$$

Proof Assume that $(\rho_j; j \in \mathcal{J} \cup \{0\})$ is a solution of (19). Let's verify that $(\rho_{uj}; uj \in (\mathcal{U} \times \mathcal{J}) \cup \{0\})$ defined by ((23) satisfy (24), i.e. for $j = 2, \ldots, J-1$

$$\rho_j \left(\lambda_{uj,0} + \lambda_{uj,u(j-1)} + \lambda_{uj,u(j+1)}\right) = \rho_0 \lambda_{0,uj} + \rho_{j-1} \lambda_{u(j-1),uj} + \rho_{j+1} \lambda_{u(j+1),uj}$$

and

$$\rho_1 \left(\lambda_{u1,0} + \lambda_{u1,u2}\right) = \rho_0 \lambda_{0,u1} + \rho_2 \lambda_{u2,u1}$$

$$\rho_J \left[\lambda_{uJ,0} + \lambda_{uJ,u(J-1)} + \sum_v \lambda_{uJ,vJ}\right] = \rho_0 \lambda_{0,uJ} + \rho_{J-1} \lambda_{u(J-1),uJ} + \sum_v \rho_J \lambda_{vJ,uJ}$$

which are clearly satisfied since $\lambda_{uj,uk} = \lambda_{jk}$ and $\sum_v \lambda_{uJ,vJ} = \sum_v \lambda_{vJ,uJ}$. □

References

1. Abou-Faycal, I. C., Trott, M. D., & Shamai, S. (2001, May). The capacity of discrete-time memoryless Rayleigh fading channels. *IEEE Transactions on Information Theory*, 1290–1301.
2. Altman, E. (2002). Capacity of multi-service CDMA cellular networks with best-effort applications. In *Proceedings of Mobicom*.
3. Anand, S., & Chockalingam, A. (2003, December). Performance analysis of voice/data cellular CDMA with SIR based admission control. *IEEE Journal of Selected Areas Communication, 21*(10).
4. Armony, M., & Bambos, N. (2003). Queueing dynamics and maximal throughput scheduling in switched processing systems. *Queueing Systems, 44*.
5. Asmussen, S. (1987). *Applied probability and queues*. New York: Springer.
6. Baccelli, F., & Brémaud, P. (2003). *Elements of queueing theory. Palm martingale calculus and stochastic recurrences*. Berlin: Springer.
7. Błaszczyszyn, B., & Karray, M. K. (2007). Performance evaluation of scalable congestion control schemes for elastic traffic in cellular networks with power control. In *Proceedings of IEEE infocom* (pp. 170–178).
8. Błaszczyszyn, B., & Karray, M. K. (2009). Dimensioning of the downlink in OFDMA cellular networks via an Erlang's loss model. In *Proceedings of European wireless*.
9. Błaszczyszyn, B., & Karray, M. K. (2009, November). Fading effect on the dynamic performance evaluation of OFDMA cellular networks. In *Proceedings of ComNet*.
10. Bonald, T., Borst, S., & Proutière, A. (2004). How mobility impacts the flow-level performance of wireless data systems. In *Proceedings of IEEE infocom* (pp. 1872–1881).
11. Bonald, T., Borst, S. C., Hegde, N., Jonckheere, M., & Proutiére, A. (2009). Flow-level performance and capacity of wireless networks with user mobility. *Queueing Systems Theory Applications, 63*(1–4), 131–164.
12. Bonald, T., & Proutière, A. (2003, September). Wireless downlink data channels: User performance and cell dimensioning. In *Proceedings of Mobicom*.
13. Bonald, T., & Proutière, A. (2006). A queueing analysis of data networks. In R. Boucherie, & N. Van Dijk (Eds.), *Queueing networks: A fundamental approach*.
14. Bordenave, C. (2004, February). *Stability properties of data flows on a CDMA network in macrodiversity*. Rapport de Recherche 5257, INRIA.
15. Borst, S. (2003). User-level performance of channel-aware scheduling algorithms in wireless data networks. In *Proceedings of IEEE Infocom*.
16. Borst, S. C., Hegde, N., & Proutiére, A. (2006). Capacity of wireless data networks with intra- and inter-cell mobility. In *Proceedings of Infocom*.
17. Borst, S. C., Hegde, N., & Proutiére, A. (2009). Mobility-driven scheduling in wireless networks. In *Proceedings of Infocom* (pp. 1260–1268).
18. Brémaud, P. (1999). *Markov chains. Gibbs fields, Monte Carlo simulation, and queues*. Berlin: Springer.
19. Chlebus, E., & Lidwin, W. (1995). Is handoff traffic really poissonian? In *Proceedings of ICUP* (pp. 348–353).
20. Cohen, J. W. (1976). *On regenerative processes in queueing theory*, Vol. 121 of Lecture Notes in Economics and Mathematical Systems. Berlin: Springer.
21. Gallager, R. G. (2008). *Principles of digital communication*. Massachusetts: Massachusetts Institute of Technology.
22. Hegde, N., & Altman, E. (2003). Capacity of multiservice WCDMA Networks with variable GoS. In *Proceedings of IEEE WCNC*.
23. Jugl, E., & Boche, H. (1999). Dwell time models for wireless communication systems. In *Proceedings of VTC* (pp. 2984–2988).
24. Karray, M. K. (2007). *Analytic evaluation of wireless cellular networks performance by a spatial Markov process accounting for their geometry, dynamics and control schemes*. PhD thesis, Ecole Nationale Supérieure des Télécommunications.
25. Liu, H., & Li, G. (2005). *OFDM-based broadband wireless networks: Design and optimization*. London: Wiley-Interscience.
26. Peppas, K., Lazarakis, F., Axiotis, D. I., Al-Gizawi, T., & Alexandridis, A. A. (2009). System level performance evaluation of MIMO and SISO OFDM-based WLANs. *Wireless Networks (Springer), 15*(7), 859–873.
27. Serfozo, R. (1999). *Introduction to stochastic networks*. New York: Springer.
28. Shin, S., Lee, K., & Kim, K. (2002). Performance of the packet data transmission using the other-cell-interference factor in DS-CDMA downlink. In *Proceedings of IEEE WCNC*.
29. Thomas, R., Gilbert, H., & Mazziotto, G. (1988). *Influence of the moving Of the mobile stations on the performance of a radio mobile cellular network*, 12–15 September.

Author Biography

Mohamed Kadhem Karray received his diploma in engineering from Ecole Polytechnique and Ecole Nationale Supérieure des Télécommunications (ENST) in 1991 and 1993, respectively. He prepared a Ph.D. thesis at ENST under the guidance of Eric Moulines and Bartek Blaszczyszyn within 2004–2007. Since 1993 he works at France Telecom R&D (Orange Labs) in France. He coauthered together with François Baccelli and Bartek Blaszczyszyn publications in scientific journals and international conferences, and together with them he hold two patents on the load control in cellular networks. His research activities aim to evaluate the performance of communication networks. his principle tools are probability and stochastic processes, and more specifically information and queueing theories. In his recent research, he shows how to articulate the tools of these two theories to build global analytical performance evaluation methods for wireless cellular networks. The methods and tools he develops are used by Orange Operator for dimensioning its networks and for several practical studies such as the effect of a reduction of the transmitted power.