Theoretically Feasible QoS in a MIMO Cellular Network Compared to the Practical LTE Performance

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Paper [4]
Theoretically feasible bit-rates

Ergodic capacity

Theoretical versus practical performance

User’s QoS calculation

Conclusion
Theoretically feasible QoS in a MIMO Cellular Network Compared to the Practical LTE Performance

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Introduction

- The objective is to build a *global analytical approach* for the evaluation of the quality of service perceived by the users in wireless cellular networks which is calibrated in some reference cases.
- QoS model for LTE (MIMO/OFDM) based on information theory, queueing theory and practical LTE system performance (3GPP simulations)
- The present work exploits previous results such as:
  - practical coding schemes performance may be evaluated by a modification of the famous Shannon’s formula $\log_2 (1 + \text{SNR})$
Theoretically feasible bit-rates

Model

- **Channel representation:**

  \[ Y_n = H_u X_{u,n} + J_n + Z_n, \quad n = 1, 2, \ldots \]

- channel input \( X_{u,n} \in \mathbb{C}^t \)
- channel output \( Y_n \in \mathbb{C}^r \)
- fading matrix \( H_u \in \mathbb{C}^{r \times t} \)
- interference

  \[ J_n = \sum_{v \neq u} H_v X_{v,n} \quad (1) \]

- noises \( Z_1, Z_2, \ldots \) are i.i.d. *circularly symmetric Gaussian* random variables
Theoretically feasible bit-rates

Model

- **Channel characteristics:**
  - flat fading
  - *quasi-static* model for fading process
  - single user detection

- **Notation:**
  - user’s base station $u$, interfering base stations $v$
  - MIMO with $t$ transmitting and $r$ receiving antennas
  - time divided into time-instants $n = 1, 2, \ldots$
  - *covariance matrix* of a random vector $X$ is denoted by $\Gamma_X = E[XX^*]$. 

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Deterministic fading: Feasible rates

- Assume fading deterministic.
- *Feasible* set of users bit-rates calculated under assumptions:
  
  - Signals of different BS are independent.
  - Signal $X_{u,n}$ transmitted by the serving BS is $CN$ with covariance matrix $\Gamma_{X_{u,n}} = \frac{P}{t} I_t$ (power equi-partition).
  - Signal $X_{v,n}$ transmitted by interfering BS $v \neq u$ is $CN$ with covariance matrix $\Gamma_{v} = \frac{P}{t} I_t$ (again power equi-partition).
  - Transmitted signals are independent from noises.
  - Signals transmitted at different time instants are independent.
Theoretically feasible bit-rates

Deterministic fading: Feasible rates

- The covariance matrix of the interference (1) equals

\[
\Gamma_J = \frac{P}{t} \sum_{v \neq u} H_v H_v^* 
\]

- Interference plus noise \( Z'_n := J_n + Z_n \) is \( \mathcal{CN} \) with covariance matrix

\[
\mathcal{N} := NI_r + \frac{P}{t} \sum_{v \neq u} H_v H_v^* 
\]

- Consequently:

\[
\Gamma_Y = \frac{P}{t} H_u H_u^* + \mathcal{N} 
\]
Theoretically feasible bit-rates

Deterministic fading: Feasible rates

- Mutual information at a given time-instant; that is
  \[ I(X_{u,n}; Y_n) = \log_2 \det \left( I_r + \frac{P}{t} H_u H_u^* N^{-1} \right) \]

- Capacity \( C = \sup I(X_{u,n}; Y_n) \); thus
  \[ C \geq \log_2 \det \left( I_r + \frac{P}{t} H_u H_u^* N^{-1} \right) \]

- The right-hand side of the above equation gives a feasible bit-rate for the considered user.

- With propagation-losses \( L_u \) and \( \{L_v\}_{v \neq u} \) one obtains:
  \[ C \geq \log_2 \det \left( I_r + \frac{P}{t} \frac{H_u H_u^* N^{-1}}{L_u} \right) \]

where
\[ N = NI_r + \frac{P}{t} \sum_{v \neq u} \frac{H_v H_v^*}{L_v} \]
Theoretically feasible bit-rates

Ergodic capacity

**Proposition**: The *ergodic capacity* $E[C]$ with respect to fading and assuming $E[H_vH_v^*] = I_r$, for all BS $v$, is given by:

$$E[C] \geq E[\log_2 \text{det} (I_r + \text{SINR}H_uH_u^*)]$$  \hspace{1cm} (4)

where the expectation is with respect to the fading $H_u$ with the serving BS and

$$\text{SINR} = \frac{(P/t)/L_u}{N + (P/t)\sum_{v \neq u} 1/L_v}$$  \hspace{1cm} (5)
Theoretically feasible bit-rates
Ergodic capacity

**Proof**: By the properties of the conditional expectation, we have $E[C] = E[E[C|H_u]]$. Equation (2) implies that

$$E[C|H_u] \geq E \left[ \log_2 \det \left( I_r + \frac{P}{t} \frac{H_u H_u^*}{L_u} \mathcal{N}^{-1} \right) \right]$$

where $\mathcal{N}$ is given by (3). Using the convexity of the function $\mathcal{N} \mapsto \log_2 \left[ \det \left( I_r + \frac{P}{t} \frac{H_u H_u^*}{L_u} \mathcal{N}^{-1} \right) \right]$ on the set of positive definite matrices of $\mathbb{C}^{r \times r}$ (see [3, Lemma II.3]) and Jensen’s inequality, we deduce that

$$E[E[C|H_u]] \geq E[\log_2 \det \left( I_r + \frac{P}{t} \frac{H_u H_u^*}{L_u} E[\mathcal{N}|H_u]^{-1} \right)]$$

$$= E[\log_2 \det \left( I_r + \frac{P}{t} \frac{H_u H_u^*}{L_u} E[\mathcal{N}]^{-1} \right)]$$

$$\geq E[\log_2 \det (I_r + H_u H_u^* \text{SINR})]$$
Theoretical versus practical performance

**AWGN**

- Consider (AWGN) SISO channel without neither fading nor interference
- *Spectral efficiency* defined as the ratio of the bit-rate by the bandwidth which equals $\log_2 \left( 1 + \frac{P/L_u}{N} \right)$
- LTE system spectral efficiency in this AWGN context is well approximated by

$$s \simeq a \log_2 \left( 1 + \frac{P/L_u}{N} \right) \quad (6)$$

- In the AWGN context, the 3GPP [2, §A.2] shows that $a \simeq 0.5$
Theoretical versus practical performance

AWGN

- Compare analytical to Orange’s simulator results:

![Graph showing theoretical and practical performance comparison](image-url)
Theoretical versus practical performance

Fading and interference

- Goal is to account for fading, MIMO and interference

\[ S(\text{SINR}, t, r) = a E \left[ \log_2 \det (I_r + H_u H_u^* \text{SINR}) \right] \]  \hspace{1cm} (7)

- What is the practical LTE spectral efficiency compared to the above analytical expression?

- Practical performance results: Orange’s simulator compliant with the 3GPP recommendation [1] in the so-called *calibration* case

- We search for some \( b \) such that

\[ s \simeq b \times S(\text{SINR}, t, r) \]  \hspace{1cm} (8)
Theoretical versus practical performance

Fading and interference

- Results of the linear fitting (8)

<table>
<thead>
<tr>
<th>MIMO</th>
<th>Scheduler</th>
<th>$b$</th>
<th>residual stand. dev.</th>
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</thead>
<tbody>
<tr>
<td>$1 \times 2$</td>
<td>RR</td>
<td>0.83</td>
<td>0.45</td>
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<tr>
<td>$1 \times 2$</td>
<td>PF</td>
<td>1.02</td>
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<td>$2 \times 2$</td>
<td>PF</td>
<td>0.67</td>
<td>0.74</td>
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<td>$4 \times 2$</td>
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<td>0.49</td>
<td>0.76</td>
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</table>
Theoretical versus practical performance

SINR

- SINR obtained by analytical approach is compared to 3GPP simulation results
- Our model is similar (details in the article) to that used by 3GPP
- Difference is the use of both, planar and toroidal network patterns and not only the planar one
Analytical versus 3GPP results for coupling gain
Analytical versus 3GPP results for SINR

![Graph showing CDF of SINR for 3GPP simulations and different network topologies](image_url)
Theoretical versus practical performance

Spectral efficiency

- We calculate the spectral efficiency corresponding to its SINR by relation (8)
- CDFs of the normalized user throughput

![CDF of normalized user throughput](image_url)
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Theoretical versus practical performance

Spectral efficiency

- **Mean** of the spectral efficiencies at the different locations

<table>
<thead>
<tr>
<th>MIMO</th>
<th>Scheduler</th>
<th>Simus</th>
<th>Analytic</th>
<th>Simus</th>
<th>Analytic</th>
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<tbody>
<tr>
<td>1 × 2</td>
<td>RR</td>
<td>1.01</td>
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<td>PF</td>
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<td>0.84</td>
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<tr>
<td>4 × 2</td>
<td>PF</td>
<td>1.54</td>
<td>1.54</td>
<td>0.95</td>
<td>1.18</td>
</tr>
</tbody>
</table>
User’s QoS calculation

- Dynamic context for VBR calls is considered for ultimate QoS evaluation
- Information theory and queueing theory time-scales
- Assuming a round robin scheduler, the peak bit-rate at each location equals the system bandwidth times the spectral efficiency at that location given by Equation (8)
- Let $\rho$ be the traffic demand (in bit/s) per cell and $\rho_c$ is the so-called critical traffic demand
- Numerical setting of the calibration case with no/high mobility of users and interference/canceled interference
Throughput per user vs cell radius for traffic density 300kbit/s/km² (typical in urban area)
Conclusion

- Analytical expression of users bit-rates which are feasible from the information theory point of view
- Throughput perceived by the users in the long run of users arrivals and departures is given
- Ultimate improvement expected from the interference cancellation is demonstrated
- Advantages of the present analytical method (much faster than pure simulations, in some aspects goes beyond the capabilities of classic simulators etc...)
- Future work
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Conclusions

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