Theoretically Feasible QoS in a MIMO Cellular Network Compared to the Practical LTE Performance

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Paper [4]

Outline

Introduction

- The objective is to build a *global analytical approach* for the evaluation of the quality of service perceived by the users in wireless cellular networks which is calibrated in some reference cases.
- QoS model for LTE (MIMO/OFDM) based on information theory, queueing theory and practical LTE system performance (3GPP simulations)
- The present work exploits previous results such as :
  - practical coding schemes performance may be evaluated by a modification of the famous Shannon's formula $\log_2(1 + \mathrm{SNR})$

Theoretically feasible bit-rates
Model

- **Channel representation :**

$$Y_n = H_u X_{u,n} + J_n + Z_n, \quad n = 1, 2, \ldots$$

  - channel input $X_{u,n} \in \mathbb{C}^t$
  - channel output $Y_n \in \mathbb{C}^r$
  - fading matrix $H_u \in \mathbb{C}^{r \times t}$
  - interference

$$J_n = \sum_{v \neq u} H_v X_{v,n} \tag{1}$$

  - noises $Z_1, Z_2, \ldots$ are i.i.d. *circularly symmetric Gaussian* random variables

Theoretically feasible bit-rates
Model

- **Channel characteristics :**
  - flat fading
  - *quasi-static* model for fading process
  - single user detection
- **Notation :**
  - user's base station u , interfering base stations v
  - MIMO with t transmitting and r receiving antennas
  - time divided into time-instants $n = 1, 2, \ldots$
  - *covariance matrix* of a random vector X is denoted by $\Gamma_X = E\left[XX^*\right].$

Theoretically feasible bit-rates

Deterministic fading : Feasible rates

- Assume fading deterministic.
- *Feasible* set of users bit-rates calculated under assumptions :
  - Signals of different BS are independent.
  - Signal $X_{u,n}$ transmitted by the serving BS is $\mathcal{CN}$ with covariance matrix $\Gamma_{X_{u,n}} = \frac{P}{t} I_t$ (power equi-partition).
  - Signal $X_{v,n}$ transmitted by interfering BS $v \neq u$ is $\mathcal{CN}$ with covariance matrix $\Gamma_v = \frac{P}{t} I_t$ (again power equi-partition).
  - Transmitted signals are independent from noises.
  - Signals transmitted at different time instants are independent.

Theoretically feasible bit-rates
Deterministic fading : Feasible rates

- The covariance matrix of the interference (1) equals

$$\Gamma_J = \frac{P}{t} \sum_{v \neq u} H_v H_v^*$$

- Interference plus noise $Z'_n := J_n + Z_n$ is $\mathcal{CN}$ with covariance matrix

$$\mathcal{N} := N I_r + \frac{P}{t} \sum_{v \neq u} H_v H_v^*$$

- Consequently :

$$\Gamma_Y = \frac{P}{t} H_u H_u^* + \mathcal{N}$$

Theoretically feasible bit-rates

Deterministic fading : Feasible rates

- Mutual information at a given time-instant ; that is

$$I\left(X_{u,n};Y_n\right)=\log_2\det\left(I_r+\frac{P}{t}H_uH_u^*\mathcal{N}^{-1}\right)$$

- Capacity $C=\sup I\left(X_{u,n};Y_n\right)$; thus

$$C\geq\log_2\det\left(I_r+\frac{P}{t}H_uH_u^*\mathcal{N}^{-1}\right)$$

- The right-hand side of the above equation gives a *feasible* bit-rate for the considered user.
- With propagation-losses L_u and $\{L_v\}_{v\neq u}$ one obtains :

$$C\geq\log_2\det\left(I_r+\frac{P}{t}\frac{H_uH_u^*}{L_u}\mathcal{N}^{-1}\right)\ , \tag{2}$$

where

$$\mathcal{N}=NI_r+\frac{P}{t}\sum_{v\neq u}\frac{H_vH_v^*}{L_v} \tag{3}$$

Theoretically feasible bit-rates

Ergodic capacity

Proposition : The *ergodic capacity* $E[C]$ with respect to fading and assuming $E[H_v H_v^*] = I_r$, for all BS v, is given by :

$$E[C] \geq E[\log_2 \det(I_r + \mathrm{SINR} H_u H_u^*)] \tag{4}$$

where the expectation is with respect to the fading H_u with the serving BS and

$$\mathrm{SINR} = \frac{(P/t)/L_u}{N + (P/t)\sum_{v \neq u} 1/L_v} \tag{5}$$

Theoretically feasible bit-rates

Ergodic capacity

Proof : By the properties of the conditional expectation, we have $E[C] = E[E[C|H_u]]$. Equation (2) implies that

$$E[C|H_u] \geq E\left[\log_2 \det\left(I_r + \frac{P}{t}\frac{H_u H_u^*}{L_u}\mathcal{N}^{-1}\right)\middle| H_u\right]$$

where $\mathcal{N}$ is given by (3). Using the convexity of the function $\mathcal{N} \mapsto \log_2\left[\det\left(I_r + \frac{P}{t}\frac{H_u H_u^*}{L_u}\mathcal{N}^{-1}\right)\right]$ on the set of positive definite matrices of $\mathbb{C}^{rxr}$ (see [3, Lemma II.3]) and Jensen's inequality, we deduce that

$$\begin{aligned} E[E[C|H_u]] &\geq E[\log_2 \det\left(I_r + \frac{P}{t}\frac{H_u H_u^*}{L_u}E[\mathcal{N}|H_u]^{-1}\right)] \\ &= E[\log_2 \det\left(I_r + \frac{P}{t}\frac{H_u H_u^*}{L_u}E[\mathcal{N}]^{-1}\right)] \\ &\geq E[\log_2 \det\left(I_r + H_u H_u^* \mathrm{SINR}\right)] \end{aligned}$$

Theoretical versus practical performance
AWGN

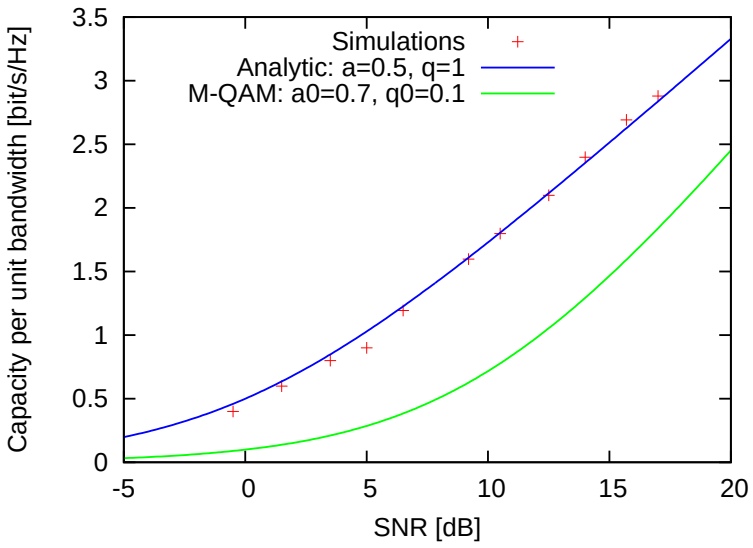
- Consider (AWGN) SISO channel without neither fading nor interference
- *Spectral efficiency* defined as the ratio of the bit-rate by the bandwidth which equals $\log_2\left(1+\frac{P/L_u}{N}\right)$
- LTE system spectral efficiency in this AWGN context is well approximated by

$$s \simeq a\log_2\left(1+\frac{P/L_u}{N}\right) \tag{6}$$

- In the AWGN context, the 3GPP [2, §A.2] shows that $a \simeq 0.5$

Theoretical versus practical performance AWGN

- Compare analytical to Orange's simulator results :



Theoretical versus practical performance
Fading and interference

- Goal is to account for fading, MIMO and interference

$$S\left(\mathrm{SINR}, t, r\right) = aE\left[\log_2 \det\left(I_r + H_u H_u^* \mathrm{SINR}\right)\right] \quad (7)$$

- What is the practical LTE spectral efficiency compared to the above analytical expression ?
- Practical performance results : Orange's simulator compliant with the 3GPP recommendation [1] in the so-called *calibration* case
- We search for some b such that

$$s \simeq b \times S\left(\mathrm{SINR}, t, r\right) \quad (8)$$

Theoretical versus practical performance
Fading and interference

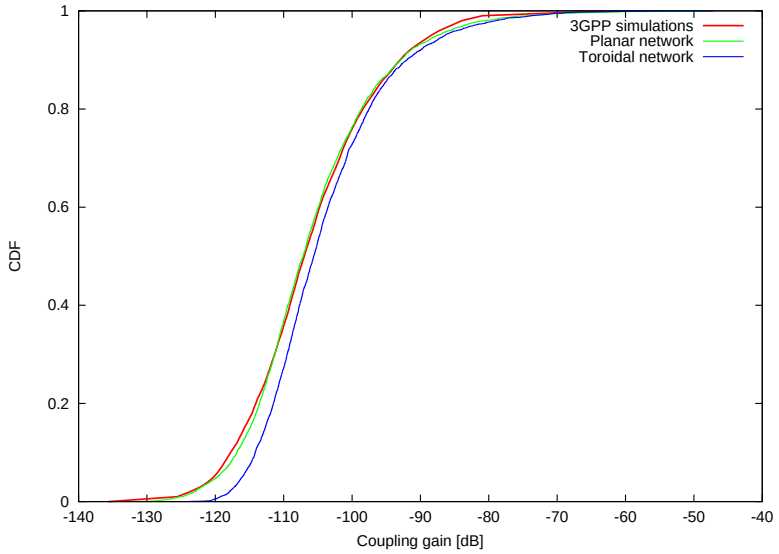
- Results of the linear fitting (8)

MIMO	Scheduler	b	residual stand. dev.
1×2	RR	0.83	0.45
1×2	PF	1.02	0.65
2×2	PF	0.67	0.74
4×2	PF	0.49	0.76

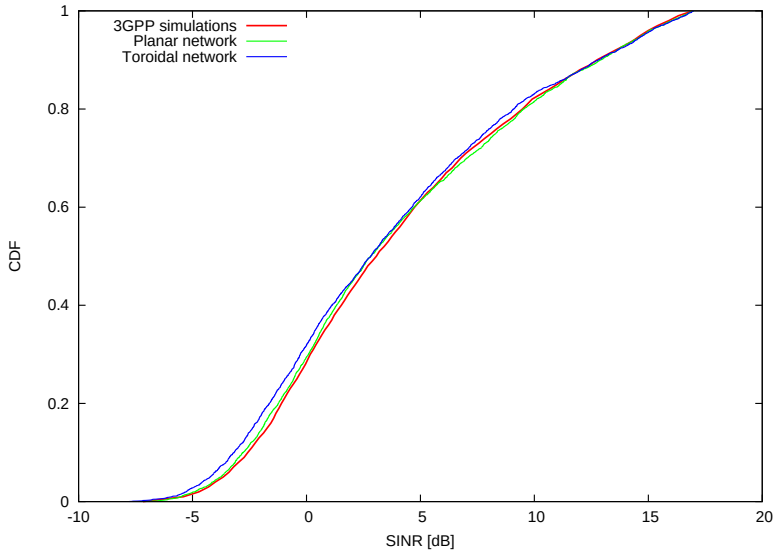
Theoretical versus practical performance
SINR

- SINR obtained by analytical approach is compared to 3GPP simulation results
- Our model is similar (details in the article) to that used by 3GPP
- Difference is the use of both, planar and toroidal network patterns and not only the planar one

Analytical versus 3GPP results for coulping gain

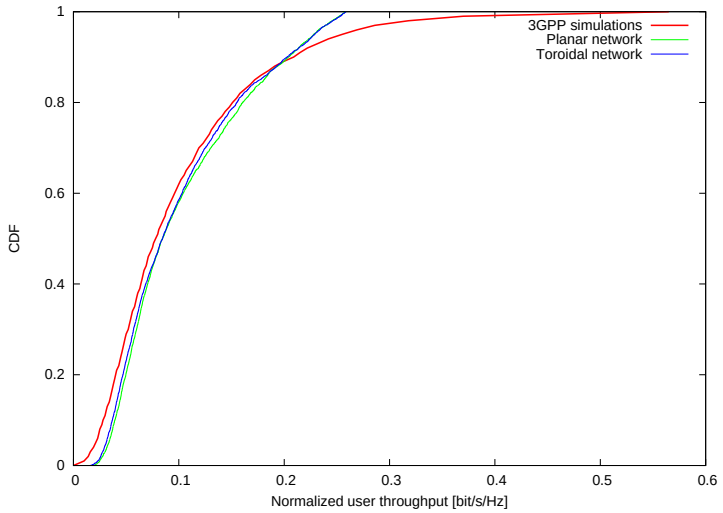


Analytical versus 3GPP results for SINR



Theoretical versus practical performance
Spectral efficiency

- We calculate the spectral efficiency corresponding to its SINR by relation (8)
- CDFs of the normalized user throughput



Theoretical versus practical performance
Spectral efficiency

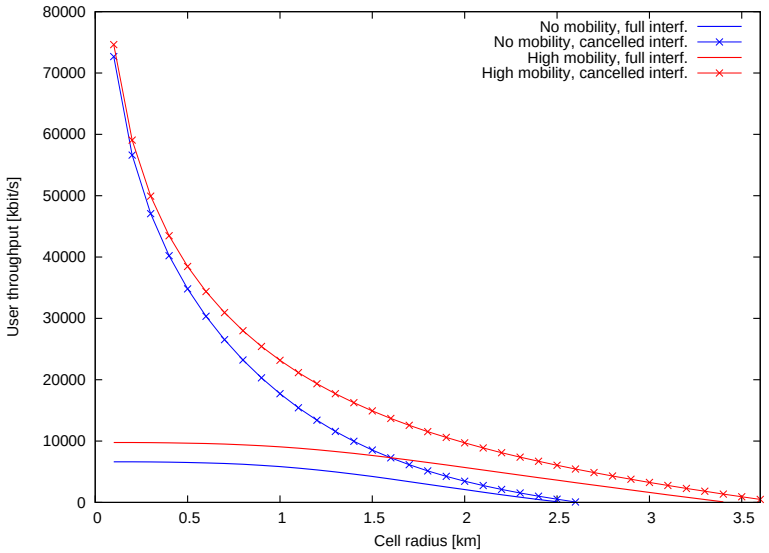
- *Mean* of the spectral efficiencies at the different locations

		Arithmetic mean		Harmonic mean	
MIMO	Scheduler	Simus	Analytic	Simus	Analytic
1×2	RR	1.01	1.00	0.50	0.69
1×2	PF	1.32	1.23	0.80	0.85
2×2	PF	1.43	1.41	0.84	1.00
4×2	PF	1.54	1.54	0.95	1.18

User's QoS calculation

- Dynamic context for VBR calls is considered for ultimate QoS evaluation
- Information theory and queueing theory time-scales
- Assuming a round robin scheduler, the peak bit-rate at each location equals the system bandwidth times the spectral efficiency at that location given by Equation (8)
- Let ρ be the traffic demand (in bit/s) per cell and ρ_{c} is the so-called *critical traffic demand*
- Numerical setting of the calibration case with no/high mobility of users and interference/canceled interference

Throughput per user vs cell radius for traffic density 300kbit/s/km^2 (typical in urban area)



Conclusion

- Analytical expression of users bit-rates which are feasible from the information theory point of view
- Throughput perceived by the users in the long run of users arivals and departures is given
- Ultimate impovement expected from the interference cancellation is demonstrated
- Adventages of the present analytical method (much faster than pure simulations, in some aspects goes beyond the capabilities of classic simulators etc...)
- Future work

Bibliography

3GPP.
TR 36.814-V900 Further advancements for E-UTRA - Physical Layer Aspects.
In *3GPP Ftp Server*, 2010.

3GPP.
TR 36.942-V830 Evolved Universal Terrestrial Radio Access (E-UTRA) - Radio Frequency (RF) system scenarios.
In *3GPP Ftp Server*, September 2010.

S.N. Diggavi and T.M. Cover.
The worst additive noise under a covariance constraint.
IEEE Trans. Inf. Theory, 47(7) :3072–3081, 2001.

M. K. Karray.
Spectral and energy efficiencies of OFDMA wireless cellular networks.
In *Proc. of IFIP Wireless Days*, October 2010.