A queueing theoretic approach to the dimensioning of wireless cellular networks serving variable bit-rate calls

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***Abstract*—The traffic demand in wireless cellular networks is increasing rapidly, especially for the data transmission. It is crucial to characterize the network parameters, such as the number of base stations, permitting to cope with this increase. Such characterization, called *dimensioning*, is the central objective of the present paper.**

Our approach consists of using results from queueing theory in order to build a rapid and accurate method to calculate the quality of service (QoS) perceived by the users. The comparison of the results of this method to those of 3GPP simulations for some LTE configurations permits to validate the accuracy of the proposed approach.

Once validated, this approach is used to solve the dimensioning problem. In doing so, we take into account the dependence between the interference and the traffic demand, and compare the results to those of the classical assumption neglecting such dependence. Therefore, the proposed approach uses the 3GPP link simulation results and goes beyond by solving rapidly QoS evaluation and dimensioning problems.

***Keywords*-Dimensioning, Traffic, Interference, Load, QoS, Cellular, Wireless**

I. INTRODUCTION

The traffic demand in wireless cellular networks is increasing rapidly and is expected to explode in the next decade. To respond efficiently to this demand the new generation of mobile cellular systems called LTE (Long Term Evolution) is developed as a successor of the currently deployed 2G (GSM) and 3G (HSDPA) systems. LTE brings significant improvements of spectral efficiency primarily using OFDM (Orthogonal Frequency Division Multiplexing) which diminishes significantly the intersymbol and intracell interference and MIMO (Multiple Input Multiple Output) antennas that exploit space/time diversity.

The deployment of such networks, frequently based on coverage conditions should now be revised to account for this traffic increase; and in particular, a densification is sometimes required. But how many sites are required to satisfy a given traffic demand with a specified quality of service target? This is the *dimensioning problem*.

The objective of the present work is to develop an approach based on queueing theory to solve this problem in an efficient and rapid way.

Manuscript received March 21, 2012; accepted January 27, 2013.

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We focus on variable bit-rate (VBR) traffic; that is users requiring some volume of date to transmit at a bit-rate which may be decided by the network. In this case, the traffic demand may be expressed in bit/s/km^2 and the quality of service in term of the throughput (in bit/s) offered to the users in the long run of arrivals and departures.

A. Related work

The dimensioning of cellular networks is often treated from a *coverage* or *static capacity* point of view. Basically one aims to assure that the bit-rate (or the SINR) of a permanent user exceeds some target value with a high probability. To do so, in [1] the cumulative distribution function (CDF) of the so-called effective SINR ('averaged' over the different OFDM subcarriers) is calculated with the help of a Gaussian approximation. Then this CDF is used to assure the coverage condition. A similar approach is adopted in [2] where other approximations for the CDF of the SINR are proposed. In these works the quality of service perceived by the users (i.e. throughput obtained by each of them) in the long run of their arrivals and departures is not calculated. This *dynamic context* is the main focus of the present paper.

The dimensioning problem in this dynamic context may, conceptually, be solved using a simulation approach such as that proposed in [3] by 3GPP (*3rd Generation Partnership Project*), a group of industrial actors. Each contributor develops his own simulation tool and compares his results to those of other contributors on some calibration cases. There are some other simulation tools (not necessarily compliant with 3GPP) such as LTE-Sim [4] developed by TelematicsLab, LTE simulator developed by University of Vien [5], [6] and LENA tool [7], [8] developed by CTTC.

The simulation approach requires such a huge amount of time that it is useless in the context of dimensioning. Indeed, calculating the users quality of service for a particular network configuration by 3GPP simulations takes up to weeks of calculation, and thus the dimensioning problem, which requires tens of such calculations, would take about a year! Analytic alternatives to the pure simulations have already been extensively proposed and studied for VBR calls. They are essentially based on queueing theory.

Information theoretic formulae are proposed in [9], [10] to approximate real link performance in a *static* context; i.e. for a given population of the users. The *dynamics* of users arrivals and departures are taken into account in [11], [12], [13] assuming that the *base stations are always transmitting their maximal power*. In this context , the *peak bit-rate* at a given location is defined as the bit-rate obtained by a user at that location assumed alone in the cell. The quality of service perceived by the users in the long run of their arrivals and departures is then calculated using multi-class processor sharing models [14], [15, Proposition 3.1]. The effect of mobility on the users' QoS is studied in [16, §4], [17].

Indeed, the base station emit only when they have at least one user to serve, and thus interference depends on the traffic of other base stations. In order to account for this dependence, the authors of [18] describe a fixed-point problem and propose to solve it iteratively.

B. Our contribution

In the present paper we build an analytical approach rather than a new simulation tool:

- We account for the dynamics of call arrivals and departures and calculate within this context the QoS perceived by users. This represents a step forward compared to the classical coverage (or static capacity) point of view.
- We continue the idea in [18] of studying the dependence of load on traffic demand by developing an analytical approach based on queueing theory.
- We validate our whole approach by comparing our result to those of 3GPP simulations [3]; and illustrate it by solving involved dimensioning problems within small computation times.

C. Paper organization

Section II shows how to calculate the QoS perceived by the users from the so-called peak bit-rates at each location. Section III gives the relation of these peak bit-rates to the SINR called link level relation and shows how to account for the dependence between the interference and the traffic demand with the help of the results of Section II. In Section IV, the link level relation is calibrated and the global approach is validated using 3GPP simulations. Moreover, we illustrate in this section our approach by solving crucial dimensioning problems.

D. Notations

We give the notations in their order of appearance in the text:

- J number of possible locations of the cell. $D = \{1, 2, \ldots, J\}$ set of locations in the cell.
- R_j peak bit-rate at each location $j \in \{1, 2, \ldots, J\}$.
- x_j number of users at location j. N total number of users in the cell.
- φ_j portion of time allocated to a user at location j.
- r_j bit-rate allocated to a user at location j.
- λ_j inverse of the inter-arrival duration at location j. μ_j inverse of the volume of data (in bits) to be transmitted at location j. $\rho_j = \lambda_j/\mu_j$ traffic demand at location j. $\rho = \sum_{j=1}^{J} \rho_j$ total traffic demand in the cell. $\lambda = \sum_{j=1}^{J} \lambda_j$ total arrival rate.
- ρ_c critical traffic demand. $\theta = \frac{\rho}{\rho_c}$ cell load.
- $\pi(\cdot)$ steady state the distribution of the configuration of users.
- $\bar{N}_j, \bar{T}_j, \bar{r}_j$ mean number of users, delay and throughput per user at location j.
- $\bar{N}, \bar{T}, \bar{r}$ mean number of users, delay and throughput per user in the cell.
- K number of mobile categories.
- $\tilde{P}$ maximal power transmitted by a base station. W total system bandwidth.
- b gap of capacity of real systems compared to the ultimate performance given by information theory.
- L_v propagation loss with base station v. SINR signal to interference and noise ratio. $\psi(\cdot)$ link performance function.
- $I(t)$ interference at time t. f interference factor.

II. User's QoS

We consider a cell comprising a finite set $\{1, 2, \ldots, J\}$ of possible *locations*. We denote by R_j the *peak bit-rate* at each location $j \in \{1, 2, \ldots, J\}$ of the cell; that is the bit-rate allocated by the base station to the user at this location assuming that: (1) the user is alone in the considered cell; and (2) the other base stations transmit at their maximal powers (this assumption will be revisited later).

We describe now the *allocation of the resources* to the different users present in the cell at a given time. Let x_j be the number of users at location j and $x = (x_1, x_2, \ldots, x_J)$ be the vector counting the number of users at each location called *configuration of the users*. Assume that the base station allocates to each user at location j a specific portion of time φ_j depending on its location, then such user gets the bit-rate

$$r_j = \varphi_j R_j \tag{1}$$

Writing that the sum of the time portions may not exceed 1; i.e. $\sum_{j=1}^{J} x_j \varphi_j \leq 1$, we get the following constraint on the bit-rates which may be allocated by the base station to the different users in its cell

$$\sum_{j=1}^{J} x_j \frac{r_j}{R_j} \leq 1 \tag{2}$$

We shall assume that each user gets an equal portion of time $\varphi_j = 1/N$ where N is the total number of users in the cell; then we deduce from Equation (1)

$$r_j = \frac{R_j}{N}, \quad j \in \{1, 2, \ldots, J\} \tag{3}$$

Remark 1: The constraint (2) (and the particular allocation (3)) may also be obtained by *multiplexing* the users in frequency or codes (or any mixture of time, frequency and code multiplexing). The only condition is that the users are served in a strict orthogonal way. Moreover, the bit-rates r_j should be understood as an average over a sufficiently long run of the multiplexing.

We now introduce the *dynamics* of call arrivals and departures. The inter-arrival times at location j are assumed to be exponentially distributed random variables with parameter λ_j

(average inter-arrival duration equals $1/\lambda_j$). The users arriving to location j require to transmit some volumes of data (in bits) which are i.i.d. random variables[1] of mean $1/\mu_j$. (We consider only locations j where user arrive with some data to transmit.) We assume that the inter-arrivals are independent from each other and so are the required volumes. We assume also independence between the inter-arrivals and the required volumes. We call $\rho_j := \lambda_j/\mu_j$ the *traffic demand* at location j and $\rho = \sum_{j=1}^J \rho_j$ the total traffic demand in the cell. We denote $\lambda = \sum_{j=1}^J \lambda_j$ the total arrival rate.

A. No mobility case

We assume in the present section that the users *don't move* during their calls. The following proposition gives the performance in the long run of the calls arrivals and departures. Denote the set of locations by $\mathbb{D} := \{1, 2, \ldots, J\}$. In order to position our problem in the queueing theory context, we may view a cell as a *queue* and a location as a *class*. In doing so, a cell may be considered as a *multi-class processor sharing* queue.

Proposition 1: The cell is stable when the traffic demand doesn't exceed a critical value; that is

$$\rho < \rho_c \tag{4}$$

where

$$\rho_c := \frac{\rho}{\sum_{j=1}^J \rho_j R_j^{-1}} \tag{5}$$

In case of stability, the steady state the distribution of the configuration of the users is

$$\pi(x) = \left(1 - \frac{\rho}{\rho_c}\right) x_{\mathbb{D}}! \prod_{j\in\mathbb{D}} \frac{(\rho_j/R_j)^{x_j}}{x_j!}, \quad x \in \mathbb{N}^{\mathbb{D}} \tag{6}$$

where $x = (x_j)_{j\in\mathbb{D}}$ is a vector counting the numbers of users in each location and $x_{\mathbb{D}} := \sum_{j\in\mathbb{D}} x_j$. Moreover, the mean number of users, the delay and the throughput per user *at a given location* $j \in \{1, 2, \ldots, J\}$ are respectively given by

$$\bar{N}_j = \frac{\rho_j}{\left(1 - \frac{\rho}{\rho_c}\right) R_j}, \quad \bar{T}_j = \frac{1}{\left(1 - \frac{\rho}{\rho_c}\right) R_j \mu_j}, \quad \bar{r}_j = \left(1 - \frac{\rho}{\rho_c}\right) R_j \tag{7}$$

and the mean number of users, the delay and the throughput per user *in the cell* at the steady state are respectively given by

$$\bar{N} = \frac{\rho}{\rho_c - \rho}, \quad \bar{T} = \frac{\rho}{(\rho_c - \rho)\lambda}, \quad \bar{r} = \rho_c - \rho \tag{8}$$

Proof: See the appendix for a detailed proof in the Markovian case; i.e., when the transmitted volumes are assumed exponentially distributed. In the more general case (when the transmitted volumes are arbitrary distributed) the proof is more involved. For the stability condition (4) and the expression (6) of the steady state distribution see [14], [15, Proposition 3.1].

The mean number of users, the delay and the throughput expressions may be obtained from [11] or by specializing [13, Example 10] to the current discrete context with no mobility. Note that the expression of the throughput per user in the cell; i.e. $\bar{r} = \rho_c - \rho$ comes exclusively (up to our knowledge) from [13, Example 10].

We give here an outline of the proof of Equations (7) and (8). The mean number of users (either in a given location or in the cell) is obtained from the expression (6) of the steady state distribution. The delays are then deduced from Little's formula [19]

$$\bar{T}_j = \frac{\bar{N}_j}{\lambda_j}, \quad \bar{T} = \frac{\bar{N}}{\lambda}$$

The throughput per user at a given location j is simply the average volume $1/\mu_j$ divided by the delay $\bar{T}_j$. It remains to show the expression of the throughput per user in the cell; i.e. $\bar{r} = \rho_c - \rho$. To do so, observe that the throughput of the whole cell at the steady state is equal to the traffic demand ρ; since at equilibrium the volumes of data incoming to and leaving the cell in the long run should be equal. The throughput per user in the cell is *defined* as the ratio of the cell throughput ρ by the average number of users; that is $\bar{r} = \frac{\rho}{\bar{N}} = \rho_c - \rho$. ■

Note that (6) may be written as follows

$$\pi(x) = \left[(1 - \rho')\, \rho'^{x_{\mathbb{D}}}\right] \left[x_{\mathbb{D}}! \prod_{j=1}^J \frac{(\rho'_j/\rho')^{x_j}}{x_j!}\right], \quad x \in \mathbb{N}^J$$

where $\rho'_j = \rho_j/R_j$ and $\rho' = \rho/\rho_c$. If follows that the distribution of the total number of users in the cell $X_{\mathbb{D}} := \sum_{j=1}^J X_j$ is the geometric distribution on $\mathbb{N}$ with parameter $1-\rho' = 1-\frac{\rho}{\rho_c}$; that is $\Pr(X_{\mathbb{D}} = n) = (1 - \rho')\rho'^n, n \in \mathbb{N}$. In particular the probability that the cell is not empty equals $\rho' = \frac{\rho}{\rho_c}$ (called *load* of the cell).

Moreover the above expression shows that, given the total number of users n, the distribution of the number of users among the different locations is *multinomial* of size (n, J) and parameters $(\rho'_1/\rho', \ldots, \rho'_J/\rho')$; this is equivalent to say that the users are assigned to classes independently to each other, with the probability ρ'_j/ρ' of a given user to be assigned to class j.

Corollary 1: With the notations of Proposition 1, if $\rho < \rho_c$ then

$$\bar{r} = \frac{\rho}{\sum_{j=1}^J \rho_j \bar{r}_j^{-1}}$$

and

$$\bar{T} = \frac{1}{\lambda} \sum_{j=1}^J \lambda_j \bar{T}_j$$

where $\lambda = \sum_{j=1}^J \lambda_j$ is the total arrival rate to the cell.

Proof: Straightforward calculations from (7) and (8). ■

The above Corollary shows that the throughput per user in the cell is the *harmonic* mean of the throughputs at the different locations pondered by the traffic demands; whereas the delay per user in the cell is the *arithmetic* mean of the delays at the different locations pondered by the arrival rates. So we should be carefully when calculating the average of the quality of service over a cell.

[1] not necessarily exponentially distributed

1) Mobile categories: A user located at a given geographic location undergoes some *radio conditions*; i.e., some specific propagation losses (due to distance, shadowing and indoor) with the different base stations in the network. Given these radio conditions, the user gets some bit-rate. The relation between the radio conditions and the bit-rate (which will be presented in more details in Section III-A) may be specific to each mobile *category*. We shall use the term *class* to designate not only the geographic location but also the specific mobile's category.

Let K be the number of mobile categories and I be the number of geographic locations, then a class j is a pair (i,k) where $i \in \{1,2,\ldots,I\}$ is a geographic location and $k \in \{1,2,\ldots,K\}$ is the mobile category. The number of classes is now $J = I \times K$. With this extended notion of class, the results of Proposition 1 obviously apply; we get in particular the expression of the throughput and delay for each class $j = (i,k)$. The following proposition gives the expressions of the throughput and delay per mobile's category but averaged over the geographic locations.

Proposition 2: Assume the stability condition 4. For a given mobile's category $k \in \{1,2,\ldots,K\}$, the throughput per user in the cell at the steady state is

$$\bar{r}_k = \frac{\sum_{i=1}^{I} \rho_{i,k}}{\sum_{i=1}^{I} \rho_{i,k} \bar{r}_{i,k}^{-1}}$$

that is the *harmonic* mean of the throughputs at the different geographic locations pondered by the corresponding traffic demands. The delay per user in the cell at the steady state is

$$\bar{T}_k = \frac{\sum_{i=1}^{I} \lambda_{i,k} \bar{T}_{i,k}}{\sum_{i=1}^{I} \lambda_{i,k}}$$

that is the *arithmetic* mean of the delays at the different geographic locations pondered by the corresponding arrival rates.

Proof: The result is obtained by specializing [13, Example 10] to the current discrete context with no mobility. ■

2) Dimensioning: We shall assume in the present section that the traffic geographic distribution

$$p_j = \frac{\rho_j}{\rho}, \quad j \in \{1,2,\ldots,J\}$$

is fixed whereas the total traffic demand in the cell ρ may vary. For example, in the uniform case $p_j = 1/J$ for all $j \in \{1,2,\ldots,J\}$. In this case, Equation (5) reads

$$\rho_{\mathrm{c}} := \left[\sum_{j=1}^{J} p_j R_j^{-1}\right]^{-1}$$

which shows that ρ_{c} is independent of the cell traffic demand ρ. Then the stability condition (4) says that the traffic demand ρ should not exceed the so-called *critical traffic* ρ_{c} which is the harmonic mean of the peak bit-rates pondered by the traffic distribution $(p_1,p_2,\ldots,p_J)$.

Fixing a target value $\bar{r}$ of the throughput per user in the cell, we deduce from (8)

$$\rho_{\mathrm{c}} - \rho = \bar{r}$$

called the *dimensioning constraint*. Indeed, the *dimensioning* consists of calculating the number of base stations per unit surface (or equivalently the cell radius) as function of traffic demand per surface unit. Since ρ and ρ_{c} are function of the cell radius, the dimensioning may be carried by solving the above equation with respect to the cell radius.

Remark 2: The above formulation of the dimensioning problem assumes that the cells have the same radius as in the case of the regular Hexagonal model. The irregularity of the network will be taken into account in future work.

We assume, without loss of generality, that the peak bit-rates are sorted in the decreasing order; that is $R_1 > R_2 > \ldots > R_J$. Fixing a target value $\bar{r}_J$ of the throughput per user in the cell border, we deduce from (7)

$$\frac{\rho}{\rho_{\mathrm{c}}} = 1 - \frac{\bar{r}_J}{R_J}$$

which may be taken as the dimensioning constraint.

Given some $q \in [0,1]$, let j_q be the q-quantile of the traffic distribution $(p_1,p_2,\ldots,p_J)$; i.e. such that

$$\sum_{j=1}^{j_q-1} p_j < q \leq \sum_{j=1}^{j_q} p_j$$

Then fixing a target value $\bar{r}_{j_q}$ of the throughput per user at location j_q, we deduce from (7)

$$\frac{\rho}{\rho_{\mathrm{c}}} = 1 - \frac{\bar{r}_{j_q}}{R_{j_q}}$$

which may be taken also as the dimensioning constraint.

Remark 3: Note that j_q is not the q-quantile of the proportion of users at the steady state $\left(\bar{N}_1/\bar{N}, \bar{N}_2/\bar{N}, \ldots, \bar{N}_J/\bar{N}\right)$ since from (7)

$$\frac{\bar{N}_j}{\bar{N}} = \frac{\rho_{\mathrm{c}}}{\rho}\frac{\rho_j}{R_j} = p_j \frac{\rho_{\mathrm{c}}}{R_j}$$

Thus we should not say that a proportion q *of the users* at the steady state would have a throughput larger than $\bar{r}_{j_q}$; but we should say that a proportion q *of the cell surface* (pondered by the traffic demand) would have a throughput larger than $\bar{r}_{j_q}$.

B. Infinite mobility

The objective is to study the effect of mobility on performance from the queueing theory point of view. The case when the average user's speed is finite and nonnull is intractable analytically. But it may be bounded by the two extreme cases of no mobility and infinite mobility since mobility improves performance as proved in [16, §4.2.2]. This motivates our study of the infinite mobility case where each user is assumed to move along all the possible locations and thus experiences all the radio conditions during his call (whereas in the no mobility case, the user undergoes a given radio condition).

We assume in the present section that the mean volume of data doesn't depend on the location; that is $\mu_j \equiv \mu$. We assume also that each user moves according to some ergodic Markov process with invariant distribution $(\sigma_1,\sigma_2,\ldots,\sigma_J)$. Moreover we assume that each user moves so fast that he receives a peak bit-rate averaged over his mobility; that is $\sum_{j=1}^{J} \sigma_j R_j$. Then the bit-rate allocation (3) is now replaced by

$$r_j \equiv r := \frac{\sum_{j=1}^{J} \sigma_j R_j}{N}, \quad j \in \{1, 2, \ldots, J\} \tag{9}$$

Proposition 3: In case of infinite mobility, the cell is stable when

$$\rho < \rho_{\mathrm{c}}$$

where

$$\rho_{\mathrm{c}} := \sum_{j=1}^{J} \sigma_j R_j \tag{10}$$

In case of stability, the mean number of users, the delay and the throughput per user in the cell at the steady state are respectively given by

$$\bar{N} = \frac{\rho}{\rho_{\mathrm{c}} - \rho}, \quad \bar{T} = \frac{\rho}{(\rho_{\mathrm{c}} - \rho)\,\lambda}, \quad \bar{r} = \rho_{\mathrm{c}} - \rho$$

Proof: See [17, Proposition 2]. ■

Note that ρ_{c} is the arithmetic mean of the peak bit-rates pondered by the mobility distribution $(\sigma_1, \sigma_2, \ldots, \sigma_J)$. Assume that the traffic demand $(\rho_1, \rho_2, \ldots, \rho_J)$ is proportional to the mobility distribution, then, since the arithmetic mean is larger than the harmonic mean, we deduce that the critical traffic with mobility is larger than the critical traffic in the no mobility case which is coherent with [16, §4.2.2].

1) Mobile categories: If there are different mobile categories, then it is natural to assume that mobility holds between the geographic locations $\{1, 2, \ldots, I\}$ but not between the mobile categories $\{1, 2, \ldots, K\}$. We assume that the mean volume of data doesn't depend on the geographic location but may depend on the mobile's category; that is

$$\mu_{i,k} \equiv \mu_k, \quad i \in \{1, 2, \ldots, I\}, k \in \{1, 2, \ldots, K\}$$

Again we assume that each user of category $k \in \{1, 2, \ldots, K\}$ moves so fast that he receives a peak bit-rate averaged over his mobility; that is

$$R_k := \sum_{l=1}^{I} \sigma_{l,k} R_{l,k}, \quad k \in \{1, 2, \ldots, K\} \tag{11}$$

Thus the bit-rate allocation is now

$$r_{i,k} \equiv r_k := \frac{\sum_{l=1}^{I} \sigma_{l,k} R_{l,k}}{N}, \quad i \in \{1, 2, \ldots, I\}, k \in \{1, 2, \ldots, K\}$$

Proposition 4: The cell is stable when

$$\rho < \rho_{\mathrm{c}} := \frac{\rho}{\sum_{k=1}^{K} \rho_k R_k^{-1}}$$

where R_k are given by (11) and

$$\rho_k := \sum_{l=1}^{I} \rho_{l,k}, \quad k \in \{1, 2, \ldots, K\}$$

is the cell traffic for category k. In case of stability, the mean number of users, the delay and the throughput per user of category $k \in \{1, 2, \ldots, K\}$ are respectively given by

$$\bar{N}_k = \frac{\rho_k}{\left(1 - \frac{\rho}{\rho_{\mathrm{c}}}\right) R_k}, \quad \bar{T}_k = \frac{1}{\left(1 - \frac{\rho}{\rho_{\mathrm{c}}}\right) R_k \mu_k}, \quad \bar{r}_k = \left(1 - \frac{\rho}{\rho_{\mathrm{c}}}\right) R_k$$

and the mean number of users, the delay and the throughput per user in the cell at the steady state are respectively given by

$$\bar{N} = \frac{\rho}{\rho_{\mathrm{c}} - \rho}, \quad \bar{T} = \frac{\rho}{(\rho_{\mathrm{c}} - \rho)\,\lambda}, \quad \bar{r} = \rho_{\mathrm{c}} - \rho$$

Proof: Observe that the present context is similar to that of Proposition 1 with the categories here in the role of the locations there and where the peak bit-rates are given now by (11). The desired results then follow from Proposition 1. ■

III. Peak bit-rates

We show now how to get the peak bit-rates $R_1, R_2, \ldots, R_J$ in a typical LTE network. Indeed, the peak bit-rate at a given location depends on the signal to interference and noise ratio (SINR) at the considered location and on the relation between the SINR and the bit-rate; i.e. the so called *link performance*.

A. Link performance

The capacity (in the asymptotic sense of information theory) of a MIMO channel with a given fading state H, is given by [20]

$$w \log_2 \left[\det\left(I_r + \mathrm{SNR} H H^*\right)\right]$$

where w is the channel bandwidth, t and r are the numbers of transmitting and receiving antennas respectively, SNR is the signal to noise power ratio per transmitting antenna, I_r is the identity matrix of dimension r and H^* designates the transpose complex conjugate of H. As observed in [21, §I], since we consider variable bit-rate traffic, we may assume that the capacity is averaged over the fading state H which gives the so-called *ergodic capacity*; that is

$$wE\left[\log_2 \left[\det\left(I_r + \mathrm{SNR} H H^*\right)\right]\right]$$

In the case of a channel with interference, it is usual to make the approximation that the above formula applies by replacing the SNR by the signal to interference and noise ratio (SINR); see [22, Equation (3.169)]. Therefore the user's bit-rate r is (approximately) given by

$$r \simeq wE\left[\log_2 \left[\det\left(I_r + \mathrm{SINR} H H^*\right)\right]\right]$$

In order to account for the performance of practical systems, we may consider the bit-rate r related to the SINR through the simulations as for example those described in [3] for LTE. Moreover, we may look for a *fitting* of the simulation results with the above analytical expression; that is we may search for the value of b such that the bit-rate r and the SINR obtained from the simulations are approximately related by

$$r \simeq bwE\left[\log_2 \left[\det\left(I_r + \mathrm{SINR} H H^*\right)\right]\right] \tag{12}$$

The parameter b in the above equation accounts for the gap of capacity of real systems compared to the ultimate performance given by information theory.

In summary, denoting by W the total system bandwidth, the peak bit-rate is given by

$$R = W \psi\left(\mathrm{SINR}\right)$$

for some function ψ which may be obtained either from information theory for the ultimate performance or from simulations of practical systems. The quantity $R/W = \psi(\mathrm{SINR})$ is called the *spectral efficiency* (expressed in bit/s/Hz). Note that the SINR in the above formula doesn't comprise fading since it has been already averaged out.

B. SINR

An LTE cellular network is composed of base stations covering some geographic zone. Each base station transmits at some power limited to some maximal value $\tilde{P}$ and assigns a *specific* portion w of the total system bandwidth W to each user.

Since fading is already averaged out at the link level, the remaining *propagation loss* comprises only the distance and shadowing. Consider a given user and let L_v be his propagation loss with base station v. We assume that each user is served by the base station (denoted by index 0) with the smallest loss; that is $L_0 = \inf\{L_v\}$. Assume moreover that each base station transmits a constant power spectral density.

We assume in the present section that each base station transmits at its maximal power $\tilde{P}$. Then the received signal power equals

$$p = \frac{w}{W}\frac{\tilde{P}}{L_0}$$

and the interference equal

$$I = \frac{w}{W}\sum_{v\neq 0}\frac{\tilde{P}}{L_v}$$

Let N be the noise power in the system bandwidth, then the SINR per transmitting antenna[2] equals

$$\mathrm{SINR} = \frac{\frac{p}{t}}{\frac{w}{W}N + \frac{I}{t}} = \frac{1}{\frac{NL_0}{t\tilde{P}} + f} \tag{13}$$

where

$$f := \sum_{v\neq 0}\frac{L_0}{L_v} \tag{14}$$

is called the *interference factor*. The SINR calculated by Equation (13) should be injected in Equation (12) to get the corresponding bit-rate.

C. Load

We assumed in the previous section that the interfering base stations transmit always at their maximal power $\tilde{P}$. Indeed a base station has not to transmit when there are no users to serve. The power transmitted by base station v is then $1\{X_v(t)\neq 0\}\tilde{P}$ where $X_v(t)$ is the number of users served by base station v at time t.

Thus the interference at time t equals

$$I(t) = \frac{w}{W}\sum_{v\neq 0} 1\{X_v(t)\neq 0\}\frac{\tilde{P}}{L_v}$$

The stability condition of the network in this case is not yet known. Nevertheless, the full activity assumption made in the previous section gives a useful lower bound of the peak bit-rates and thus a lower bound of the critical traffic demand. We shall make now a heuristic development which leads to an approximation of the actual peak bit-rates and critical traffic.

[2]See [22, Equation (3.169)].

Invoking the law of large numbers, we may approximate the interference as follows

$$\begin{aligned} I(t) &\simeq \frac{w}{W}\sum_{v\neq 0} E[1\{X_v(t)\neq 0\}]\frac{\tilde{P}}{L_v} \\ &= \frac{w}{W}\sum_{v\neq 0}\Pr(X_v(t)\neq 0)\frac{\tilde{P}}{L_v} \\ &= \frac{w}{W}\sum_{v\neq 0}\frac{\rho}{\rho_{\mathrm{c}}}\frac{\tilde{P}}{L_v} \end{aligned}$$

where for the third equality we use the observation following Proposition 1. Then the SINR equals now

$$\mathrm{SINR} = \frac{1}{\frac{NL_0}{t\tilde{P}} + \frac{\rho}{\rho_{\mathrm{c}}}f}$$

and the corresponding peak bit-rate equals

$$R = W\psi\left(\frac{1}{\frac{NL_0}{t\tilde{P}} + \frac{\rho}{\rho_{\mathrm{c}}}f}\right)$$

Equations (5) and (10) show that the critical traffic ρ_{c} is a function of the peak bit-rates which are them selves functions of the critical traffic as shown in the above equation. Thus ρ_{c} may be obtained by solving a fixed-point problem. For example, in the case of infinite mobility Equation (10) implies

$$\rho_{\mathrm{c}} = E[R] = E\left[W\psi\left(\frac{1}{\frac{NL_0}{t\tilde{P}} + \frac{\rho}{\rho_{\mathrm{c}}}f}\right)\right] \tag{15}$$

where expectation is with respect to a user distributed according to the mobility invariant distribution σ. Once the above fix-point problem is solved, the ratio

$$\theta := \frac{\rho}{\rho_{\mathrm{c}}} \tag{16}$$

is called the *load* of the system.

Remark 4: Note that the load depends on the traffic demand, so we can not consider these two parameters as independent inputs when evaluating the users QoS.

Remark 5: The above queueing analysis is carried for a typical cell of a network composed of multiple cells assumed statistically equivalent. Indeed, the interference between the different cells is taken into account through the interference factor (14) and the resolution of the fixed-point problem (15).

IV. Numerical results

The 3GPP (*3rd Generation Partnership Project*) is a group of industrial actors which specifies and evaluates the radio interface of the LTE wireless cellular system. In particular, The 3GPP defines in [23, Table A.2.1.1-3] and [3, Table A.2.2-1] a particular numerical setting for the *calibration* (comparison of the results) of the simulation tools of the different contributors to the project. We shall consider this calibration setting as the starting point for our numerical study.

As simulation results, we consider the results of tools which are compliant with 3GPP approach [3] (Orange's simulator developed in C++ being one of them). The average (as well confidence intervals at 20% and 80%) of the results of the different contributors to 3GPP will be plotted and compared to our analytical approach (implemented in Matlab).

We begin by describing the subset of the parameters in [23, Table A.2.1.1-3], [3, Table A.2.2-1] which are used in our analytical calculations. The frequency carrier equals 2GHz; the distance loss model is $L = 128.1 + 37.6 \times \log_{10}(r)$ [in dB] (where r is in km); the penetration loss equals 20dB and the shadowing is centered and log-normally distributed with standard deviation 8dB. The antenna pattern in the horizontal plan is $A(\varphi) = -\min\left(12\left(\varphi/\varphi_{3\mathrm{dB}}\right)^2, A_{\mathrm{m}}\right)$ where $\varphi_{3\mathrm{dB}} = 70°, A_{\mathrm{m}} = 20$dB. The system bandwidth equals $W = 10$MHz; the noise power is $N = -95$dBm and the base station transmission power equals $P = 60$dBm (including antenna gain).

A. Calibration in a static context

The objective of the present section is to *calibrate* the analytical formula of link performance derived from information theory with 3GPP simulation results in a *static context*; i.e. the users are permanently present in the network[3]. This context is called *full buffer* traffic model in [3, §A.2.1.3].

We consider a MIMO system with 2 antennas at the transmitter and 2 antennas at the receiver and proportional fair scheduler. Figure 1 shows the spectral efficiency (defined as the ratio of the peak bit-rate to the system bandwidth) as function of SINR obtained from 3GPP simulations. The fitting (12) leads to $b = 0.34$ and the corresponding analytical curve is represented in Figure 1. This figure shows that the analytical curve approximates the average tendency of the empirical data obtained from simulations.

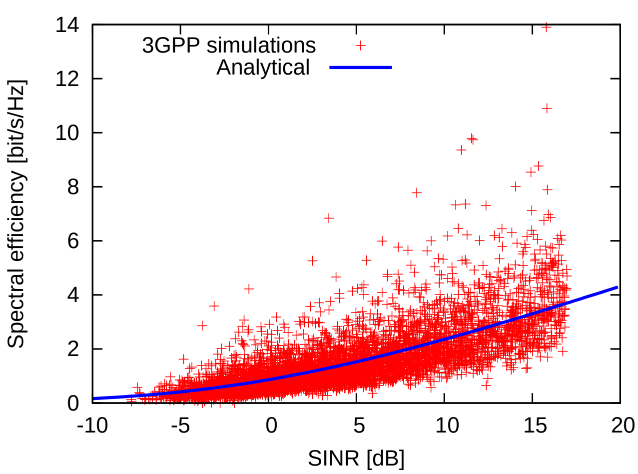

Fig. 1. MIMO 2×2: Comparison of the simulation results and the analytical expression (12)

For the following calculations, the network is modelled by 36 hexagons (6×6) on a tore (to avoid the border effects). A site with three sectorial antennas is placed at the center of each hexagon. The inter-site distance is 500m (urban area). 3600 random user locations are generated uniformly in the network. Each user is served by the base station giving the lowest *propagation-loss* (accounting for the distance and shadowing effects as well as the antenna pattern).

[3]Hence the base stations are always transmitting at their maximal power.

Our geometric model of the network is different from that proposed by 3GPP [3, Table A.2.2-1] which is planer (whereas our model is toroidal) and composed of a central hexagon surrounded by two rings of hexagons which gives a total of 19 hexagons (whereas we have $36 = 6 \times 6$ hexagons). We choose the toroidal model in order to make the roles of the different base stations (and cells) in the network completely symmetric. Indeed, Figure 2 shows that our model and that of 3GPP are similar in terms of the resulting distribution of the SINR which is the basis of the subsequent calculation of the QoS perceived by the users.

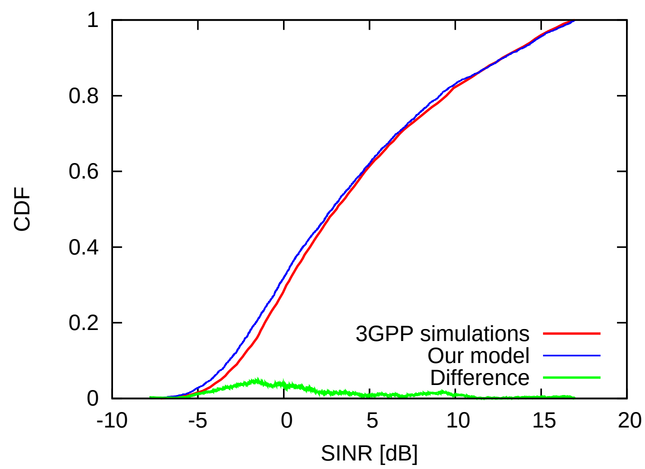

Fig. 2. Comparison of the cumulative distribution functions (CDF) of the SINR resulting from our model and from 3GPP simulations [3, Figure A.2.2-1 (right)]

B. Validation in a dynamic context

The aim is to compare the results of queueing approach described in Section III-C to those of 3GPP simulations in a *dynamic context*; i.e. calls arrive and depart from the network and each base station transmits only when it has at least one user to serve. This context is called *FTP traffic model* in [3, §A.2.1.3.1].

Figure 3 gives the load as function of traffic demand per cell resulting from 3GPP simulations and from the queueing approach. For 3GPP simulations, the load is calculated as the fraction of time where a base station has at least one user to serve. The average of the simulation results of the different 3GPP contributors as well the confidence intervals at 20% and 80% are plotted. For the queueing approach, the load is calculated by Equation (16) where the critical traffic ρ_c is solution of the fixed-point problem (15). We observe in Figure 3 that the two loads calculated by these two methods are close, except when the queueing load is close to 1. Indeed in this case, the system is at its limit of stability and therefore the time averages converge very slowly to their ergodic limits [24, p.114]. This explains why the 3GPP simulations are too time consuming at high loads (up to 3 weeks of calculation) and also the gap between the simulation and queueing loads in Figure 3. To get the curves in Figure 3, the computing time for the 3GPP simulations is several weeks whereas it is about 1 minute for the queueing approach.

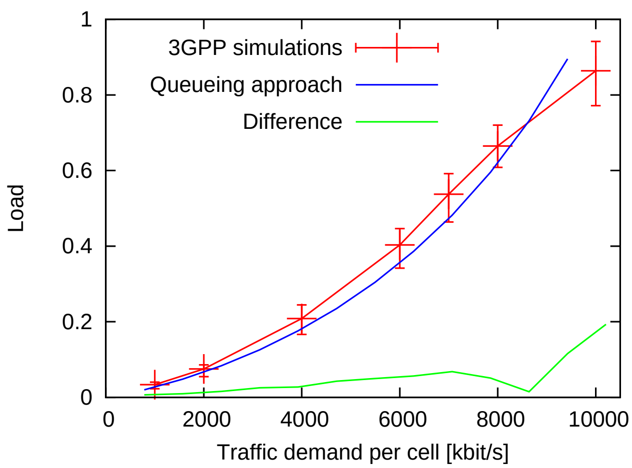

Fig. 3. Load versus traffic demand per cell

Definition 1: The following different load situations are considered in conjunction of the *queueing approach*:

- *Adapted load*: A base station transmits only when it has at least one user to serve.
- *Full load*: Base stations are always transmitting at their maximal power.
- *Null load*: Interference is assumed completely cancelled. This corresponds to a cell in isolation.

Figure 4 gives the mean user throughput as function of traffic demand for the different load situations described in Definition 1 and for 3GPP simulations. In this latter case, the mean user throughput is obtained by averaging the users throughput over the whole simulation time.

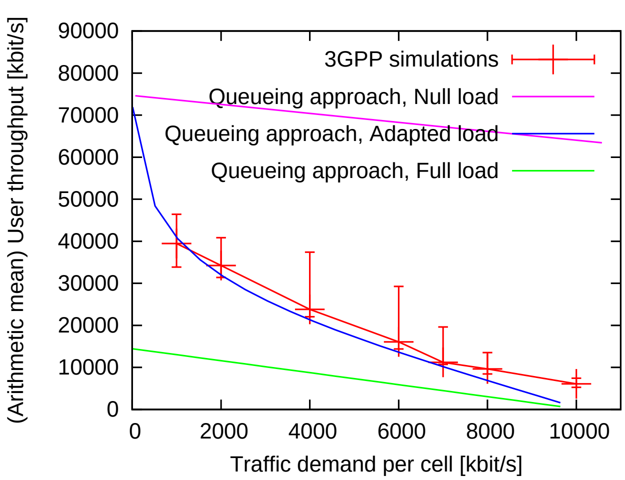

Fig. 4. Mean user throughput versus traffic demand per cell

One can observe that the curve obtained from 3GPP simulations and the one derived from queueing approach with adapted load are close, except for the highest value of traffic demand (which corresponds to a load close to 1). This gap is due to the slow convergence rate of the 3GPP simulations discussed above. On the other hand, as expected, the curves for the full and adapted load converge for the highest value of the traffic demand since this corresponds to the limit of stability of the network (when user throughput vanishes). Going backward with the values of traffic demand one the difference between these two curves increases up to factor 5. Finally, the null and adapted load curves have the same value for the smallest traffic demand and diverge as the traffic demand increases.

Figure 5 gives the 95% quantile of user throughput as function of traffic demand for 3GPP simulations and the queueing approach. Observe that the quantiles of 3GPP simulations are smaller than those of queueing approach with adapted load; nevertheless the two curves have the same tendency. This is related to the fact that peak bit-rates of 3GPP simulations are more dispersed than the analytical ones as shown in Figure 1. On the other hand, the null and full load curves agree with the adapted load one for the smallest and the highest values of traffic demand, respectively.

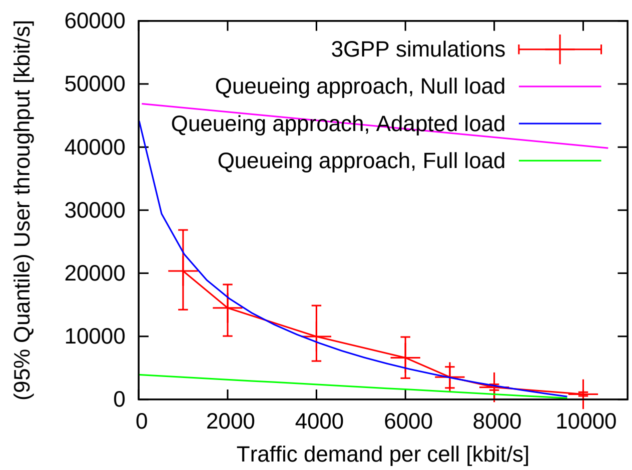

Fig. 5. 95% quantile of user throughput versus traffic demand

C. Applications

We aim now to illustrate the queueing approach on other practical problems.

Figure 6 shows mean user throughput as function of cell radius for traffic demand densities equal to 0.1 and 10Mbit/s/km^2 and different load situations (see Definition 1). The mean user throughput $\bar{r}$ is calculated by (8) where the critical traffic ρ_c is solution of the fixed-point problem (15).

As expected, for each value of the traffic demand, the curves are in the decreasing order for respectively the null, adapted and full load situations. Moreover, observe that the null and adapted load curves are close to each other for the small value of traffic demand since in this case the interference is too small. Contrarily, for higher value of traffic demand,

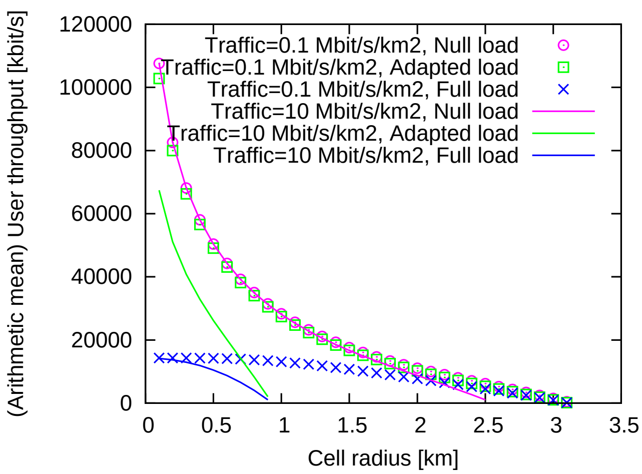

Fig. 6. Mean user throughput as function of the cell radius for different load situations

interference is significant so that adapted load curve deviates from the null load one. Moreover, as observed previously, the adapted and full load curves converge for the limit of stability of the network (when user throughput vanishes). The computing time to get Figure 6 is of some minutes, whereas it would require several weeks for 3GPP simulations.

Figure 7 shows mean user throughput as function of cell radius for different traffic demand densities for adapted load situation. As expected each curve is decreasing and ultimately vanishes for some critical value of cell radius corresponding to the stability limit of the network. Additionally, when the traffic increases, the curves decrease and the critical cell radius decreases rapidly. On the other hand, the curves for traffic demands of 10 and 100kbit/s/km^2 are close to that of null traffic up to the cell radius of 2km which shows that noise is preponderant against interference.

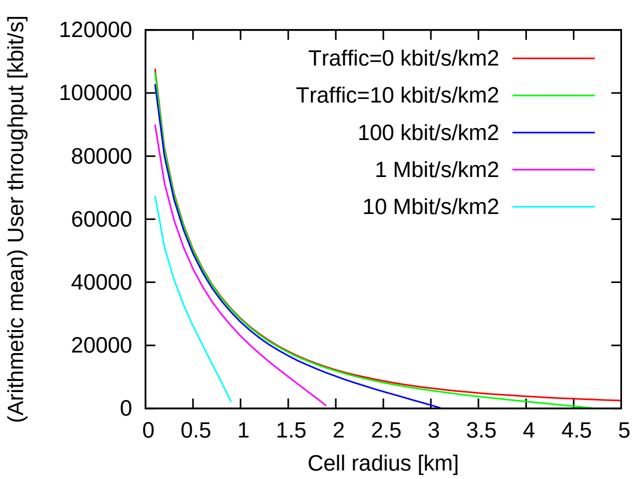

Fig. 7. Mean user throughput as function of the cell radius for different traffic demand densities (adapted load)

We give now the numerical solution of the dimensioning problem in terms of the cell radius which is more appealing than the number of base stations per unit surface (note, however, that the latter is inversely proportional to the square of the former). Figure 8 shows cell radius versus traffic demand density for two target arithmetic means of the user throughput equal to 1 and 10Mbit/s for the three load situations described in Definition 1. For the smaller user throughput, the three curves are close to each other whereas for the larger throughput they differ significantly from each other. The adapted load curve lies between the null and full load ones; and meets each of them for low and high traffic, respectively. This is due to the fact that when traffic increases, the network evolves from noise-limited to interference-limited regime.

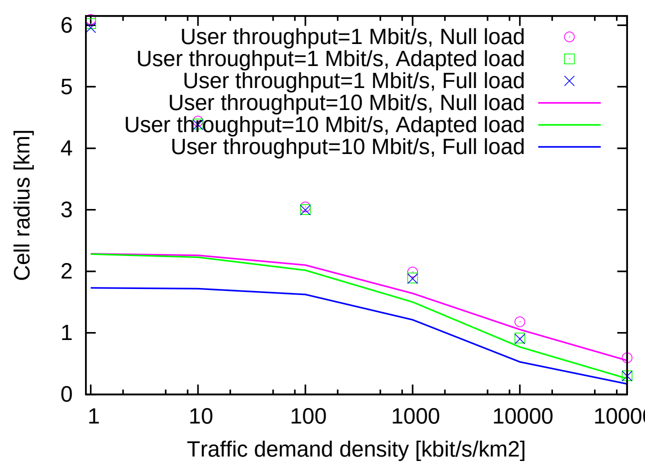

Fig. 8. Cell radius versus traffic demand density for mean user throughput 10^4kbit/s

Figure 9 shows cell radius versus traffic demand density for different target values of the arithmetic mean of the user throughput. As expected, the cell radius is decreasing with the traffic demand and with the user throughput. Note that, for the three largest user throughputs, the curve comprises a stationary part corresponding to a coverage constraint and a decreasing part corresponding to a capacity constraint.

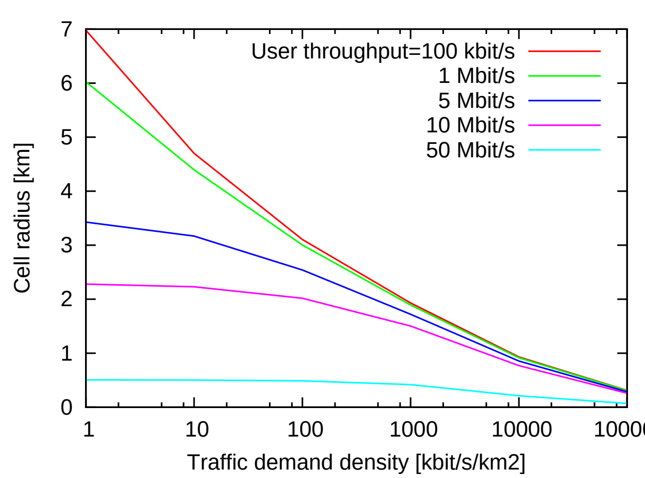

Fig. 9. Cell radius versus traffic demand density for different mean user throughputs

Figures 8 and 9 are obtained in few minutes by the analytical approach, whereas they would require several months for 3GPP simulations.

V. Conclusion

We develop an approach for the dimensioning of wireless cellular networks serving variable bit-rate calls using results from queueing theory. In doing so, we account for the dependence of the interference on the traffic demand.

The proposed approach comprises three steps. Firstly, the link performance formula given by information theory is *calibrated* with 3GPP simulation results. Secondly, the QoS perceived by the users, in the long run of their arrivals and departures, given by queueing theory is compared to that calculated by 3GPP simulations. The good match between these two types of results *validates* the proposed approach. This approach permits to reduce the computing time from weeks for 3GPP simulations to minutes. Finally, the dimensioning problem is solved and the situations where the dependence of the interference on the traffic demand has to be taken into account are identified. Therefore, the proposed approach uses the 3GPP link simulation results and goes beyond by solving rapidly QoS evaluation and dimensioning problems.

The dimensioning for streaming traffic as well as mixing such traffic with variable bit-rate calls are important axes for future work.

Appendix
Proof of Proposition 1 in the Markovian case

Assume that the transmitted volumes are *exponentially distributed*. In this particular case, the process $\{X(t);t\geq 0\}$ describing the number of users of the different classes is a continuous-time Markov process with discrete state space $\mathbb{N}^{\mathbb{D}}$ and admits the following generator

$$\begin{cases} q(x,x+\varepsilon_j)=\lambda_j, & x\in\mathbb{N}^{\mathbb{D}} \\ q(x,x-\varepsilon_j)=\mu_j R_j \frac{x_j}{x_{\mathbb{D}}}, & x\in\mathbb{N}^{\mathbb{D}}, x_j>0 \end{cases} \tag{17}$$

where ε_j designates the vector of $\mathbb{N}^{\mathbb{D}}$ having coordinate 1 at position j and 0 elsewhere and $x_{\mathbb{D}}:=\sum_{j\in\mathbb{D}}x_j$ designates the total number of users in the queue. It is easy to see that the process $\{X(t);t\geq 0\}$ is regular [25, p.337] and irreducible [25, p.357] and that it admits as invariant measure

$$\alpha(x)=x_{\mathbb{D}}!\prod_{j\in\mathbb{D}}\frac{\left(\rho_j'\right)^{x_j}}{x_j!},\quad x\in\mathbb{N}^{\mathbb{D}} \tag{18}$$

where $\rho_j':=\lambda_j/(\mu_j R_j)=\rho_j/R_j$. If $\rho':=\sum_{j=1}^{J}\rho_j'<1$ then $\sum_{x\in\mathbb{N}^{\mathbb{D}}}\alpha(x)=\frac{1}{1-\rho'}$, indeed

$$\begin{aligned}\frac{1}{1-\rho'}&=\sum_{n=0}^{\infty}\rho'^n\\&=\sum_{n=0}^{\infty}\left(\sum_{j\in\mathbb{D}}\rho_j'\right)^n\\&=\sum_{n=0}^{\infty}\sum_{x\in\mathbb{N}^{\mathbb{D}}:x_{\mathbb{D}}=n}n!\prod_{j\in\mathbb{D}}\frac{\rho_j'^{x_j}}{x_j!}\\&=\sum_{x\in\mathbb{N}^{\mathbb{D}}}x_{\mathbb{D}}!\prod_{j\in\mathbb{D}}\frac{\left(\rho_j'\right)^{x_j}}{x_j!}=\sum_{x\in\mathbb{N}^{\mathbb{D}}}\alpha(x)\end{aligned}$$

We deduce that if $\rho'<1$ then the process $\{X(t);t\geq 0\}$ admits $\pi=(1-\rho)\alpha$ as invariant distribution; and hence this process is t-positive recurrent [25, p.357]. We deduce from (18) that the invariant distribution is

$$\pi(x)=(1-\rho')\,x_{\mathbb{D}}!\prod_{j\in\mathbb{D}}\frac{\left(\rho_j'\right)^{x_j}}{x_j!},\quad x\in\mathbb{N}^{\mathbb{D}}$$

Let $X=(X_1,X_2,\ldots,X_J)$ be the vector counting the number of users of each class at the steady state, and let $X_{\mathbb{D}}:=\sum_{j\in\mathbb{D}}X_j$ be the total number of users in the queue. The vector X has π as distribution, then, for $n\in\mathbb{N}$,

$$\begin{aligned}P(X_{\mathbb{D}}=n)&=\sum_{x\in\mathbb{N}^{\mathbb{D}}:x_{\mathbb{D}}=n}\pi(x)\\&=(1-\rho')\sum_{x\in\mathbb{N}^{\mathbb{D}}:x_{\mathbb{D}}=n}n!\prod_{j\in\mathbb{D}}\frac{\left(\rho_j'\right)^{x_j}}{x_j!}\\&=(1-\rho')\,\rho'^n\end{aligned}$$

which is the geometric distribution on $\mathbb{N}$ with parameter $1-\rho'=1-\rho/\rho_{\mathrm{c}}$ where ρ_{c} is given by (5). The mean number of users is

$$\bar{N}:=E[X_{\mathbb{D}}]=\frac{\rho'}{1-\rho'}=\frac{\rho}{\rho_{\mathrm{c}}-\rho}$$

From Little's formula [19] the expected delay, denoted $\bar{T}$, equals

$$\bar{T}=\frac{E[X_{\mathbb{D}}]}{\lambda}=\frac{\rho}{(\rho_{\mathrm{c}}-\rho)\lambda}$$

At the steady state the queue throughput equals the traffic demand ρ. The throughput per user is *defined* as the ratio of the above queue throughput by the average number of users; that is

$$\bar{r}=\frac{\rho}{E[X_{\mathbb{D}}]}=\rho_{\mathrm{c}}-\rho$$

For a given class $j\in\mathbb{D}$,

$$\begin{aligned}\bar{N}_j&:=E[X_j]\\&=\sum_x x_j\pi(x)\\&=\sum_{x:x_j\neq 0}(1-\rho')\,x_{\mathbb{D}}!x_j\prod_{i\in\mathbb{D}}\frac{\left(\rho_i'\right)^{x_i}}{x_i!}\\&=\sum_{x'}(1-\rho')\,(x_{\mathbb{D}}'+1)\,x_{\mathbb{D}}'!\prod_{i\in\mathbb{D}}\frac{\left(\rho_i'\right)^{x_i}}{x_i'!}\\&=\sum_{x'}(x_{\mathbb{D}}'+1)\,\pi(x')\\&=\rho_j'E[X_{\mathbb{D}}+1]=\frac{\rho_j}{\left(1-\frac{\rho}{\rho_{\mathrm{c}}}\right)R_j}\end{aligned}$$

where for the fourth equality we introduce the vector x' related to x as follows

$$x_i'=\begin{cases}x_i & i\neq j\\ x_i-1 & i=j\end{cases}$$

From Little's formula the expected delay, denoted $\bar{T}_j$, equals

$$\bar{T}_j=\frac{E[X_j]}{\lambda_j}=\frac{1}{\left(1-\frac{\rho}{\rho_{\mathrm{c}}}\right)R_j\mu_j}$$

The expected throughput of class j, denoted $\bar{r}_j$, is the average required volume μ_j^{-1} divided by the expected delay, that is

$$\bar{r}_j = \frac{\mu_j^{-1}}{\bar{T}_j} = \left(1 - \frac{\rho}{\rho_c}\right) R_j$$

Acknowledgement 1: We thank S.E. Elayoubi, S. Jeux, L. Labeguerie (Orange Labs) and B. Błaszczyszyn (INRIA) for their encouragements to carry the work described in the present paper and for useful discussions.

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