# Evaluation and comparison of resource allocation strategies for new streaming services in wireless cellular networks

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***Abstract*—Since a long time wireless networks have been offering two types of services: variable bit-rate such as mail and constant bit-rate such as voice. Recently they also offer service for streaming calls such as mobile television. The objective of the present paper is to study the problem of allocating (and sharing) the resources for this new type of calls and to compare the performance of different allocation strategies.**

**The allocation strategy implemented in real networks is an *iterative time-sharing* strategy. We build an *explicit formulation* of this strategy which permits not only a faster implementation, but also an *analytical evaluation* of its performance. Moreover we describe an *optimal* allocation strategy consisting of assigning the resources in priority to the least consuming ones.**

**A cornerstone to evaluate and compare the different allocation strategies is the so-called *outage probability* defined for each location as the probability that a user in this location doesn't get his required bit-rate. We compare the efficiency of different numerical methods to calculate it; and show that the *inverse Laplace method* is rapid and efficient. We also calculate the increase of the outage probability of the time-sharing strategy *compared* to the optimal one. Finally, we validate a method permitting to *calculate analytically* the average of the outage probabilities over the different locations.**

***Index Terms*—Resource allocation, Streaming, Wireless cellular networks, Performance, Analytical**

## I. Introduction

The wireless cellular networks offer different kinds of services which may be classified into two main categories. Variable bit-rate (VBR) connections (such as mail, ftp) aim to transmit some given volume of data at a bit-rate which may be decided by the network. Constant bit-rate (CBR) calls (such as voice, video conferences) require a constant bit-rate for some duration.

Recently, wireless cellular networks began offering *streaming* services to their users; e.g. mobile television and streaming video. The present paper focuses on sharing the available resources among the streaming users; or equivalently, the allocation of the bit-rates to users.

The objective is to evaluate and compare the performance of different *resource allocation strategies* for these new streaming services.

### A. Related work

The resources (power and frequency bandwidth) in wireless cellular networks are limited. Sharing those resources between the users has already been extensively studied for CBR and VBR services.

In order to guarantee the requested bit-rates for CBR calls, a new user is admitted only if he doesn't break out the resource constraint; otherwise, the arriving call is *blocked* (i.e. definitely lost). The authors of [1], [2], [3] propose efficient admission conditions. The performance of the admission conditions is evaluated in terms of the *blocking probability*. A particular effort has been made to build conditions whose blocking probability may be evaluated analytically [4], [5], [6].

VBR connections are systematically admitted (no blocking) and they transmit at a bit-rate which may be decided/modified by the network which shares the resources fairly. The counterpart is that the bit-rate of each user decreases when their number increases. Thus the *delay* may increases drastically when the traffic demand increases. This increase may be evaluated analytically by using the multi-class *processor sharing* model [7], [6]. The particular case of a HSDPA (High Speed Downlink Packet Access) network is studied in several papers such as [8], [9] and [10].

The new streaming services [11] have characteristics mixing the properties of the two previous services. Streaming calls are always admitted as VBR; but they require a fixed bit-rate as CBR [12], [13]. The counterpart of admitting systematically all streaming calls, is that some of them may be temporarily interrupted. The choice of which calls should be interrupted is called *allocation strategy*. To our knowledge the evaluation and comparison of different allocation strategies for streaming calls in wireless cellular networks has not already been studied.

### B. Paper organization

The paper is organized as follows. We describe the model for streaming call in wireless cellular networks and formulate the resource sharing problem in Section II. A practical allocation strategy as well as a theoretically optimal one are described in Section III. We present three calculating methods for the performance of these allocation strategies in Section IV. Finally, we present our numerical results in Section V.

## II. Model description

We assume there are $J$ possible users locations indexed by $j \in \{1, 2, ....., J\}$. For each location $j$, let $R_j$ be the peak bit-rate (that is the bit-rate given to a user at this location if

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he is alone in the cell), $r_j$ be the target bit-rate and $X_j$ be the number of users at the considered location. The collection of the number of users is denoted by $X = \{X_j\}_{j=1,\ldots,J}$. Moreover, let

$$\varphi_j = \frac{r_j}{R_j}$$

which is called the *requested resource* at location $j$. We may assume without loss of generality that the requested resources are sorted in the increasing order; that is $\varphi_1 \le \varphi_2 \le \ldots \le \varphi_J$.

Since the resources are limited, only some users may be served. More specifically, we assume a resource constraint in the following form: for each cell

$$\sum_{j=1}^{J} X_j \varphi_j \le 1 \tag{1}$$

If the above constraint is not satisfied, then only a subset of the users can get bit-rates equal to their target values; such users are then said to be *satisfied*. The selection of these users is called *allocation strategy*.

The *satisfaction* at location $j$, denoted by $Y_j$, equals 1 if the users at location $j$ are satisfied; and 0 otherwise. The satisfactions $\{Y_j\}_{j=1,\ldots,J}$ are random variables since they depend on the users numbers $\{X_j\}_{j=1,\ldots,J}$ which are them selves random. We denote by $\mathcal{F}_j$ the event $\{Y_j = 1\}$; that is

$$\mathcal{F}_j = \{Y_j = 1\}$$

and call it *j-satisfaction set*. Its complementary, denoted by $\mathcal{F}'_j$, is called *j-outage set*.

We shall assume that the number of users in the different locations $X_1, X_2, \ldots, X_J$ are independent Poisson random variables with respective means $\rho_1, \rho_2, \ldots, \rho_J$ (which represent the traffic demands expressed in Erlang). We define the *satisfaction probability* at location $j$ by

$$P_{\text{sat}}(j) = P(Y_j = 1) = P(\mathcal{F}_j)$$

and the *outage probability* by

$$P_{\text{out}}(j) = P(Y_j = 0) = P(\mathcal{F}'_j)$$

Within this context, we shall address two problems:

1) What is the allocation strategy *currently implemented* in real networks and is it *optimal*?
2) How to *calculate* the outage probabilities?

## III. Allocation strategies

We now present different allocation strategies for streaming calls.

### A. Optimal strategy

The optimal strategy consists of allocating the resources in priority to the less consuming users; that is the users are selected in the increasing order of their resource consumptions $\varphi_1, \varphi_2, \ldots$ until the limit given by the constraint (1). In other terms, given the number of users configuration $X = \{X_j\}_{j=1,\ldots,J}$, let $J(X)$ be defined by

$$\sum_{k=1}^{J(X)} x_k \varphi_k \le 1, \quad \text{and} \quad \sum_{k=1}^{J(X)+1} x_k \varphi_k > 1$$

Then the users located at $j \le J(X)$ are served and those located at $j > J(X)$ are interrupted. We deduce that the $j$-satisfaction set of this strategy is

$$\mathcal{F}_j = \left\{ \sum_{k=1}^{j} X_k \varphi_k \le 1 \right\}$$

*Remark 1:* Note that $\mathcal{F}_J \subset \mathcal{F}_{J-1} \subset \cdots \subset \mathcal{F}_1$, then $P_{\text{sat}}(j)$ is decreasing with $j$ and $P_{\text{out}}(j)$ is increasing with $j$. In particular, $P_{\text{out}}(j) \le P_{\text{out}}(J)$; that is

$$P_{\text{out}}(j) \le P\left( \sum_{k=1}^{J} X_k \varphi_k > 1 \right) \tag{2}$$

*Remark 2: Sorting the resource requests.* If the resource requests $\varphi_1, \varphi_2, \ldots$ are not sorted in the increasing order, then it is not immediate to define the outage probability at each location. Indeed, we need first to sort the resource requests; that is find a permutation $\sigma$ of $\{1, 2, \ldots, J\}$ such that $\varphi_{\sigma(1)} \le \varphi_{\sigma(2)} \le \ldots \le \varphi_{\sigma(J)}$. Then, we may define the *outage probability* at location $\sigma(j)$ by

$$P_{\text{out}}(\sigma(j)) = P\left( \sum_{k=1}^{\sigma(j)} X_{\sigma(k)} \varphi_{\sigma(k)} > 1 \right)$$

Practically, we may introduce the sorted resource requests $\tilde{\varphi}_j = \varphi_{\sigma(j)}$ and the corresponding traffic demands $\tilde{\rho}_j = \rho_{\sigma(j)}$; then calculate the associated $\tilde{P}_{\text{out}}(j)$ which is precisely $P_{\text{out}}(\sigma(j))$. Note moreover that if one needs only the maximal outage probability, i.e. $P_{\text{out}}(\sigma(J))$, then there is no need to sort the resource requests since $\sum_{k=1}^{\sigma(J)} X_{\sigma(k)} \varphi_{\sigma(k)} = \sum_{k=1}^{J} X_k \varphi_k$.

### B. Time-sharing strategy

This allocation strategy is based on *sharing time* between the users *iteratively* as it is the case in current HSDPA (High Speed Downlink Packet Access) and LTE (Long Term Evolution) networks.

*1) Iterative description:* In order to describe it, we consider an iteration index $n = 1, 2, \ldots$ and we denote by $b_j(n)$ the bit-rate and by $Y_j(n)$ the satisfaction at the $n$th iteration at location $j$. Since a user is satisfied if he gets a bit-rate larger than his target value, we deduce that

$$Y_j(n) = 1\{b_j(n) \ge r_j\}$$

The bit-rates $\{b_j(n)\}$ are calculated recursively as follows.

At the first iteration, we share time equally between all the users. Then each user gets $1/\sum_{j=1}^{J} X$ fraction of the time. Thus users at location $j$ get the bit-rate

$$b_j(1) = R_j / \sum_{j=1}^{J} X$$

If all the users are satisfied, then we stop the calculation. Otherwise, we assign to each satisfied user located at $j$ a bit-rate $b_j(2) = r_j$ and a corresponding time portion

$$\frac{r_j}{R_j} = \varphi_j, \quad \text{if } Y_j(1) = 1$$

The remaining time, $1 - \sum_{i:Y_i(1)=1} X_i \varphi_i$ (where the two points ':' mean 'such that'), is shared among the unsatisfied users. Thus the bit rate of an unsatisfied user $j$ equals

$$b_j(2) = \frac{R_j}{\sum_{i:Y_i(1)=0} X_i} \left( 1 - \sum_{i:Y_i(1)=1} X_i \varphi_i \right), \quad \text{if } Y_j(1) = 0$$

More generally, at iteration $n$, the bit rates are given by

$$b_j(n)=\begin{cases} r_j & \text{if } Y_j(n-1)=1 \\ \frac{R_j\left(1-\sum_{i:Y_i(n-1)=1}X_i\varphi_i\right)}{\sum_{i:Y_i(n-1)=0}X_i} & \text{otherwise}\end{cases}$$

We stop when the satisfactions are not changed from one iteration to the following; i.e.

$$Y_j(n)=Y_j(n-1),\quad \forall j\in\{1,2,3,.....,J\}$$

*2) Explicit formulation:* We now build an explicit description of the ultimate (iteration) result of the time sharing strategy.

*Proposition 1:* The (ultimate) $j$-satisfaction set of the time-sharing strategy is

$$\mathcal{F}_j=\left\{\left(\sum_{k=1}^{j-1}X_k\varphi_k\right)+\left(X_j+....+X_J\right)\varphi_j\le 1\right\}$$

*Proof:* It is enough to prove that the ultimate satisfactions $\{Y_j\}_{j=1,....,J}$ given by the time sharing strategy satisfy

$$Y_j=1\Leftrightarrow\left(\sum_{k=1}^{j-1}X_k\varphi_k\right)+\left(X_j+....+X_J\right)\varphi_j\le 1 \qquad (3)$$

Since $\left\{\varphi_j\right\}_{j=1...J}$ are sorted in the increasing order, there exists a localization $j_0\in\{1,2,.....,J\}$ so that

$$Y_1=Y_2=Y_3=Y_{j_0-1}=1 \text{ and } Y_{j_0}=Y_{j_0+1}=....=Y_J=0$$

- We begin by proving the direction '$\Leftarrow$' in Equation (3). Since $Y_{j_0}=0$, then

$$\frac{R_{j_0}}{X_{j_0}+....+X_J}\left(1-\sum_{k=1}^{j_0-1}X_k\varphi_k\right)<r_{j_0}$$

which implies

$$\sum_{k=1}^{j_0-1}X_k\varphi_k+\left(X_{j_0+}....+X_J\right)\varphi_{j_0}>1 \qquad (4)$$

Now, for any $l\ge j_0$, we have $\varphi_l-\varphi_{j_0}\ge 0$, thus we deduce that

$$\left(\sum_{k=1}^{l-1}X_k\varphi_k+\left(X_l+...+X_J\right)\varphi_l\right)-\left(\sum_{k=1}^{j_0-1}X_k\varphi_k+\left(X_{j_0}+....+X_J\right)\varphi_{j_0}\right)\ge 0$$

Thus, we have

$$\left(\sum_{k=1}^{l-1}X_k\varphi_k+\left(X_l+...+X_J\right)\varphi_l\right)\ge\left(\sum_{k=1}^{j_0-1}X_k\varphi_k+\left(X_{j_0}+....+X_J\right)\varphi_{j_0}\right)$$

Combining the above inequality and (4), we get

$$\left(\sum_{k=1}^{l-1}X_k\varphi_k+\left(X_l+...+X_J\right)\varphi_l\right)>1$$

This achieves the proof of the direction '$\Leftarrow$' in Equation (3) (using the fact that $A\Rightarrow B$ is equivalent to $\text{not}\,(B)\Rightarrow\text{not}\,(A)$).

- We now prove the direction '$\Rightarrow$' in Equation (3). Consider some $j\le j_0$. Since the locations $1,2,\ldots,j-1$ are satisfied, thus

$$\sum_{k=1}^{j-1}X_k\varphi_k\le 1$$

Consider the portion of time left free by users located at $1,2,\ldots,j-1$; that is

$$1-\sum_{k=1}^{j-1}X_k\varphi_k$$

Divide the above portion of time equally between the users at locations $j,j+1,\ldots,J$. The users at location $j$ get the bit rate

$$\frac{R_j}{X_j+...+X_J}\left(1-\sum_{k=1}^{j-1}X_k\varphi_k\right)$$

Since $Y_j=1$, the above bit-rate should be larger than $r_j$, that is

$$\frac{R_j}{X_j+...+X_J}\left(1-\sum_{k=1}^{j-1}X_k\varphi_k\right)\ge r_j$$

A simple algebraic manipulation shows that the above inequality implies the right hand side of (3), which finishes the proof of the proposition.

■

*Proposition 2:* In the same conditions as the previous proposition, the ultimate bit-rates assigned by the time-sharing strategy are

$$b_j=\begin{cases} r_j & \text{if } Y_j=1 \\ \frac{R_j\left(1-\sum_{k:Y_k=1}X_k\varphi_k\right)}{\sum_{k:Y_k=0}X_k} & \text{otherwise}\end{cases}$$

*Proof:* If $Y_j=1$, the bit rate is evidently the target value $r_j$. Consider now a location $j$ such that $Y_j=0$. Consider the portion of time left free by satisfied users; that is

$$1-\sum_{k:Y_k=1}X_k\varphi_k$$

Divide the above portion of time equally between the unsatisfied users. Then users at location $j$ get the bit rate

$$\frac{R_j}{\sum_{k:Y_k=0}X_k}\left(1-\sum_{k:Y_k=1}X_k\varphi_k\right)$$

which finishes the proof. ■

### C. Other allocation strategies

An alternative service policy to those described above consists of serving the users in the decreasing order of their resource requests $\varphi_J,\varphi_{J-1},\ldots$. The $j$-satisfaction set of this allocation strategy is

$$\mathcal{F}_j=\left\{x:\sum_{k=j}^{J}x_k\varphi_k\le 1\right\}$$

Clearly, the upper bound (2) holds again true for this strategy and also for the time-sharing strategy. In fact, this bound holds true for a large class of service policies. For this reason, we will focus on its calculation in the numerical section.

## IV. Calculation methods

We will now describe the methods permitting to calculate the outage probability or equivalently the satisfaction probability. We will consider the optimal strategy, but the results may easily be extended to the other allocation strategies described in Section III.

### A. Monte Carlo simulations

The usual Monte Carlo method consists of generating $N$ independent realizations $X^{(1)}, X^{(2)}, \ldots, X^{(N)}$ of the random vector $X$ and estimate the outage probability as follows

$$\hat{P}_{\text{out}}(j) = \frac{1}{N}\sum_{n=1}^{N} 1\left\{X^{(n)} \in \mathcal{F}'_j\right\} \tag{5}$$

where $1\{\cdot\}$ designates the indicator function. The classical concept of confidence intervals permits to calculate the error of the above estimate as function of $N$. This will be detailed in the numerical section and used to determine the number $N$ of experiments that guarantee a given accuracy.

### B. Laplace transform inversion

Note that

$$\begin{aligned} P_{\text{sat}}(j) &= P(X \in \mathcal{F}_j) \\ &= P\left(\sum_{k=1}^{j} X_k\varphi_k \leq 1\right) = F_{S_j}(1) \end{aligned}$$

where $F_{S_j}$ is the cumulative distribution function (CDF) of the random variable $S_j = \sum_{k=1}^{j} X_k\varphi_k$. There is no explicit expression of $F_{S_j}$, but fortunately, we will show that its Laplace transform has a closed form. Then we will calculate $F_{S_j}$ by inverting numerically its Laplace transform.

*Lemma 1:* Let $S$ be a random variable with values in $\mathbb{R}_+$ admitting a probability density function (PDF). Then the Laplace transform of the CDF $F_S(s) = P(S \leq s)$ is related to that of $S$ by the following relation

$$\mathcal{L}_{F_S}(\theta) = \frac{1}{\theta}E\left[e^{-\theta S}\right], \quad \theta > 0$$

*Proof:* Let $f_S$ be the PDF of the random variable $S$. For $\theta > 0$, we have

$$\begin{aligned} \mathcal{L}_{F_S}(\theta) &= \int_0^\infty e^{-\theta s} F_S(s)\, ds \\ &= \int_0^\infty e^{-\theta s}\left(\int_0^s f_S(y)\, dy\right) ds \\ &= \int_0^\infty e^{-\theta s}\left(\int_0^\infty f_S(y)\, 1_{\{y \leq s\}} dy\right) ds \\ &= \int_0^\infty f_S(y)\left(\int_0^\infty e^{-\theta s} 1_{\{y \leq s\}} ds\right) dy \\ &= \int_0^\infty f_S(y)\frac{e^{-\theta y}}{\theta} dy = \frac{1}{\theta}E\left[e^{-\theta S}\right] \end{aligned}$$

where the inversion of the integrals in the forth equality is due to Fubini theorem [14, p.148] and the fact that

$$\begin{aligned} &\int_0^\infty \int_0^\infty \left|e^{-\theta s} f_S(y)\, 1_{\{y \leq s\}}\right| dyds \\ &= \int_0^\infty \int_0^\infty e^{-\theta s} f_S(y)\, 1_{\{y \leq s\}} dyds \\ &= \frac{1}{\theta}E\left[e^{-\theta S}\right] \leq \frac{1}{\theta} < \infty \end{aligned}$$

■

*Lemma 2:* Let $X_1, X_2, \ldots, X_j$ be independent Poisson random variables with respective means $\rho_1, \rho_2, \ldots, \rho_j$; and $\varphi_1, \varphi_2, \ldots, \varphi_j$ be given positive constants. The Laplace transform of the random variable $S_j = \sum_{k=1}^{j} X_k\varphi_k$ is given by

$$E\left[e^{-\theta S_j}\right] = \exp\left[\sum_{k=1}^{j} \rho_k\left(e^{-\theta\varphi_k} - 1\right)\right], \quad \theta > 0 \tag{6}$$

*Proof:* See [15, Proposition 1.2.2]. ■

Using the above two lemmas, we deduce that the Laplace transform of the CDF of $S_j$ is given by

$$\mathcal{L}_{F_{S_j}}(\theta) = \frac{1}{\theta}\exp\left[\sum_{k=1}^{j} \rho_k\left(e^{-\theta\varphi_k} - 1\right)\right]$$

Then $P_{\text{sat}}(j) = F_{S_j}(1)$ may be retrieved by inverting the above Laplace transform by using Hoog et al. [16] algorithm implemented by Hollenbeck [17] in Matlab.

### C. Gaussian approximation

We approximate the CDF of $S_j$ with that of a Gaussian random variable with mean $m = E[S_j]$ and variance $\sigma^2 = \text{Var}[S_j]$. This leads to the following approximation of the satisfaction probability

$$P_{\text{sat}}(j) = P(S_j \leq 1) \simeq \Phi\left(\frac{1-m}{\sigma}\right)$$

where $\Phi$ is the Gaussian cumulative distribution function $\Phi(z) = 1/\sqrt{2\pi}\int_{-\infty}^{z} e^{-t^2/2}dt = \left[1 + \text{erf}\left(z/\sqrt{2}\right)\right]/2$. The expressions of the expectation and variance of $S_j$ are given in the following lemma.

*Lemma 3:* Let $X_1, X_2, \ldots, X_j$ be independent Poisson random variables with respective means $\rho_1, \rho_2, \ldots, \rho_j$; and $\varphi_1, \varphi_2, \ldots, \varphi_j$ be given positive constants. The moments of the random variable $S_j = \sum_{k=1}^{j} X_k\varphi_k$ are given by

$$E[S_j] = \sum_{k=1}^{j} \rho_k\varphi_k, \quad \text{Var}[S_j] = \sum_{k=1}^{j} \rho_k\varphi_k^2$$

*Proof:* Differentiating (6) with respect to $\theta$, we get,

$$E\left[-S_j e^{-\theta S_j}\right] = -\left(\sum_{k=1}^{j} \rho_k\varphi_k e^{-\theta\varphi_k}\right)\exp\left[\sum_{k=1}^{j} \rho_k\left(e^{-\theta\varphi_k} - 1\right)\right]$$

which at $\theta = 0$ gives the desired result for $E[S_j]$. Differentiating the above relation again with respect to $\theta$ at $\theta = 0$, we get

$$\begin{aligned} E\left[S_j^2\right] &= \sum_{k=1}^{j} \rho_k\varphi_k^2 + \left(\sum_{k=1}^{j} \rho_k\varphi_k\right)^2 \\ &= \sum_{k=1}^{j} \rho_k\varphi_k^2 + E[S_j]^2 \end{aligned}$$

from which we deduce the desired expression for $\text{Var}[S_j]$. ■

### D. Averaging over the locations

We now show how to average the outage probabilities over the different locations.

*1) Pondering by the users number:* The average outage probability pondered by the users number is given by

$$\tilde{P}_{\text{out}} = E\left[\frac{\sum_{j=1}^{J} X_j 1\left\{X \in \mathcal{F}'_j\right\}}{\sum_{j=1}^{J} X_j}\right] \tag{7}$$

This average can only be calculated by Monte Carlo simulations.

*2) Pondering by the traffic:* We assume that the set of outage probabilities $\{P_{\text{out}}(j)\}_{j=1\ldots J}$ has already been computed using one of the three methods described above. The average outage probability pondered by the traffic is given by

$$\bar{P}_{\text{out}} = \frac{\sum_{j=1}^{J} \rho_j P_{\text{out}}(j)}{\sum_{j=1}^{J} \rho_j} \tag{8}$$

## V. Numerical results

Our numerical calculations are made on data measured in a real network.

### A. Model specification

The main inputs for our model are the number of locations and the peak bit-rates at each of these locations. We get these inputs from measurements in a real HSDPA network in inner Paris as we explain now. The available bandwidth is 10MHz.

We firstly measure the signal to interference and noise ratio (SINR) at a large number of locations covering sufficiently the inner Paris. The distribution of the SINR is then deduced from these measurements and sampled into 30 values, say

$$(\text{SINR}_j, p_j), \quad j \in \{1, 2, \ldots.., 30\} \tag{9}$$

where $p_j$ designates the probability of the sample $\text{SINR}_j$ (of course $\sum_{j=1}^{30} p_j = 1$). We assume that the collection $\{\text{SINR}_j\}_{j=1\ldots30}$ is representative of the values of SINR in a typical cell of the considered network. Thus the number of locations in our model is precisely $J = 30$.

Each location $j \in \{1, 2, \ldots.., J\}$ has a specific $\text{SINR}_j$ and we calculate the corresponding peak bit-rate $R_j$ from a table which is beforehand generated by link simulations.

In order to account for the distribution in (9), the traffic demand per cell $\rho$ is distributed over the different locations as follows

$$\rho_j = \rho \times p_j$$

We shall consider different values of the traffic demand per cell $\rho = 1, 2, \ldots, 15$Erlang. The users require a bit-rate of 256Kbits/s. The simulations are made with a Java tool.

### B. Monte Carlo accuracy

The accuracy of the Monte Carlo estimate given by Equation (5) may be deduced from the concept of confidence intervals which we recall here briefly. If we make $N$ experiments, then with probability, say $\alpha = 0.99$, the estimation error does not exceed

$$\varepsilon = \frac{2.58}{\sqrt{N}}\sigma \tag{10}$$

where $\sigma$ is the standard deviation of the random variable $1\left\{X \in \mathcal{F}'_j\right\}$. (The above numerical value comes from the fact for a standard normal random variable $\mathcal{N}$ we have $P(|\mathcal{N}| \leq 2.58) = 0.99$.) Unfortunately $\sigma$ is itself unknown since it is related to the outage probability by $\sigma^2 = P_{\text{out}}(j)[1 - P_{\text{out}}(j)]$. Using the bound $\sigma^2 \leq \frac{1}{4}$, we deduce from (10) that

$$\varepsilon \leq \frac{2.58}{2\sqrt{N}}$$

We fix $\varepsilon = 10^{-3}$ and calculate the number of experiments from the above equation $N = 1.7 \times 10^6$.

### C. Results

*1) Accuracy of the outage probability estimates:* We aim to evaluate the precision of the Laplace inversion and the Gaussian approximation compared to the Monte Carlo method (which is taken as a reference).

Figure 1 represents the maximum outage probability; i.e. $P_{\text{out}}(J)$, calculated by each of these methods as function of the traffic demand per cell. We observe a good fit between the Laplace inversion and Monte Carlo, whereas the Gaussian approximation presents a visible gap. To confirm this observation, we plot in Figure 2 the estimation error defined as the difference of the reference outage probability (calculated by Monte Carlo) minus each of the two estimates. In order to improve the visibility we magnify the errors by factors of 1000 and 10 respectively for the Laplace inversion and the Gaussian approximation. We observe that the error of the Gaussian approximation is about $10^{-1}$ whereas the error of the Laplace inversion is about $10^{-3}$. Recall that the estimation error of Monte Carlo is $\varepsilon = 10^{-3}$ which equals the error observed for Laplace inversion. We deduce that the precision of Laplace inversion method is at least the same as that of Monte Carlo simulations (and perhaps better but this is not in the scope of the present work).

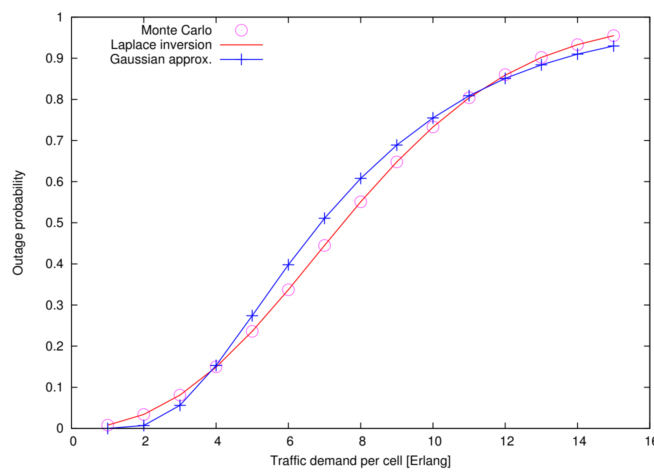


Fig. 1. Maximum outage probability calculated by the different methods.

Note that we have not specified the allocation strategy used in Figure 1 since the maximum outage probability $P_{\text{out}}(J)$ is the same for both the optimal and the time-sharing strategy.

We consider now the optimal strategy and plot the corresponding average outage probability $\bar{P}_{\text{out}}$ pondered by traffic (see Equation (8)) in Figure 3. We observe that the accuracy of the Gaussian approximation is improved; but this is due to averaging the errors over the different locations and may not be considered as a proof of Gaussian approximation accuracy. A similar result is obtained for the time-sharing allocation strategy.

The computing times to generate the curves in Figure 3 are 2s, 8s and 4h respectively for the Gaussian approximation, the Laplace inversion and the Monte Carlo method. Note the important gain of computing time of the Laplace inversion compared to Monte Carlo (reduction by a factor of about 1000). On the other hand, note that the Laplace inversion

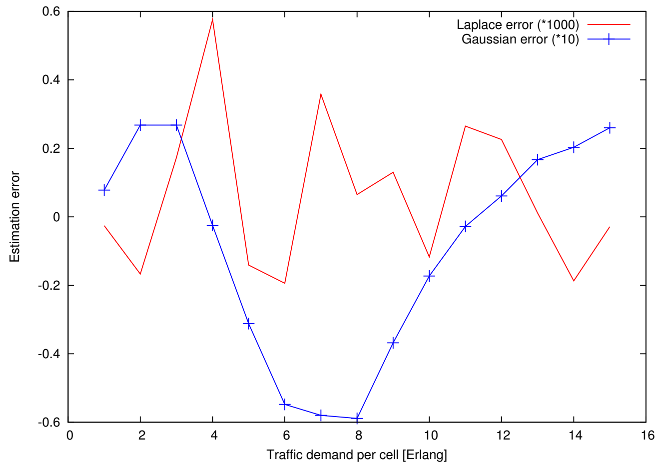


Fig. 2. Estimation error of the Laplace inversion and Gaussian approximation.

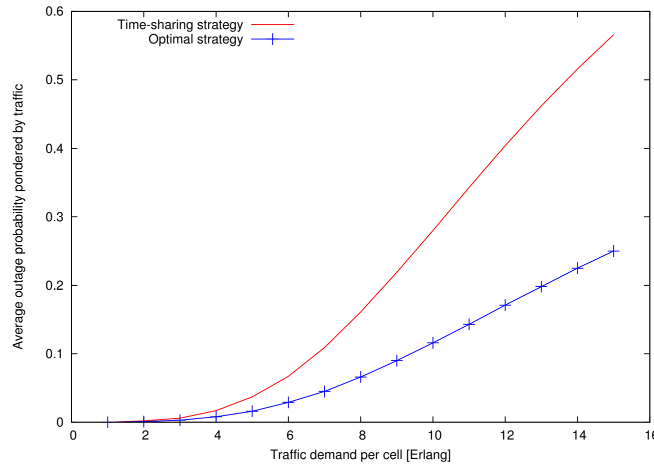


Fig. 4. Comparison of different allocation strategies.

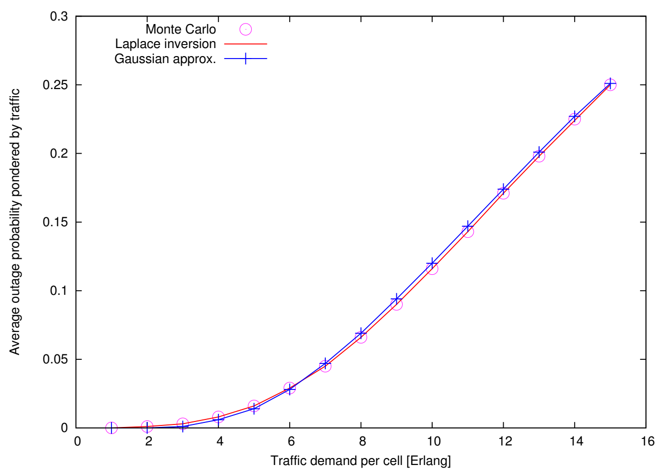


Fig. 3. Accuracy for the average outage probability estimates.

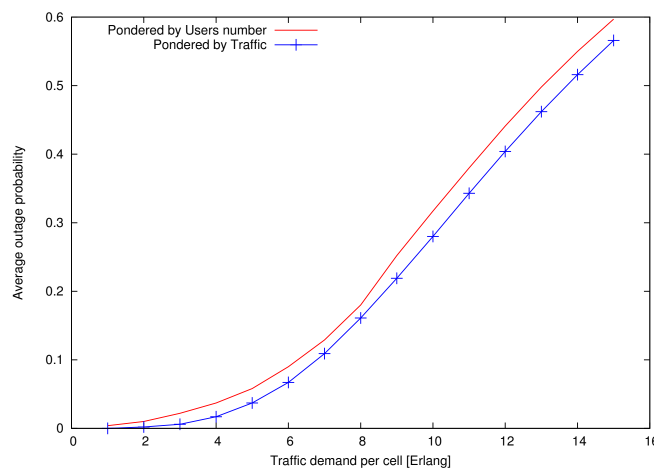


Fig. 5. Comparison of the averaging methods for the time-sharing strategy.

computing time is only about $4$ times larger than that of the Gaussian approximation.

*2) Comparison of the allocation strategies:* We compare the average outage probabilities $\bar{P}_{\text{out}}$ of the optimal and the time-sharing strategies in Figure 4. We observe that the optimal strategy gives a significantly less outage (better performance) than the time-sharing one.

*3) Comparison of the averaging methods:* In Figure 5 we compare the average outage probability pondered by traffic, that is $\bar{P}_{\text{out}}$, and by the number of users, that is $\tilde{P}_{\text{out}}$ for the time-sharing strategy. We observe that $\bar{P}_{\text{out}}$ underestimates $\tilde{P}_{\text{out}}$ but the gap remains reasonably small; not exceeding $4 \times 10^{-2}$. (We obtain a similar result for the optimal strategy.)

Our calculations are based on inputs from a real HSDPA network. We made also calculations on a LTE network with regular hexagonal architecture and theoretical inputs (SINR from usual propagation models, then peak bit-rates from information theory). We don't present these results in the paper since they are similar to those presented above for real network's inputs and the conclusions remain the same.

## VI. Conclusion

We build an *explicit formulation* of the *iterative time-sharing* allocation strategy implemented in real networks. This formulation permits not only a faster implementation, but also an *analytical evaluation* of its performance. Moreover we describe an *optimal* allocation strategy consisting of allocation the resources in priority to the least consuming ones.

The *outage probability* is a key parameter permitting to evaluate and compare the performance of the different allocations strategies. We compare the efficiency of three numerical methods to calculate it: Gaussian approximation, inverse Laplace transform and Mote Carlo simulations. Our numerical experiments show that the Gaussian approximation may be used for a first rough estimation of the outage probability (error of about $10^{-1}$); but its is *recommended to use the inverse Laplace method* if one seeks a *significantly better precision* (error not exceeding $10^{-3}$) with a moderate computing time (reduction by a factor of about $1000$ compared to Monte Carlo).

On the other hand, the comparison of the different allocation strategies shows that the optimal strategy gives a *significantly better performance* (less outage) than the time-sharing one.

Finally, in order to get a completely *analytical* method for calculating the average outage probability, we have to consider the average pondered by the traffic. This brings a gap compared to the average pondered by the number of users, but this gap remains moderate (less than $4 \times 10^{-2}$).

An interesting continuation of the present work is to consider the dynamics of call arrivals and departures and attempt to calculate the number and durations of the calls interruptions.

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